

Hint on Last Problem of Problem Set 2

In the last problem of Problem Set 2, you will need to calculate the phase shifts δ_0 and δ_1 to lowest nonvanishing order in k and k' . This will involve calculating the quantity β_ℓ given on p. 28 of the posted notes. For small ka and $k'a$ (the regime of interest), you need to expand $j_\ell(x)$ and $n_\ell(x)$ for small values of the arguments. To get a nonzero answer, you need to expand $j_\ell(x)$ through the *second* nonvanishing term in x . Only the first term is given in Shankar. The limiting form through second order is

$$j_\ell(x) \sim \frac{x^\ell}{(2\ell + 1)!!} \left[1 - \frac{x^2}{2(2\ell + 3)} + \dots \right]. \quad (1)$$

(You will need only the lowest-order asymptotic form for $n_\ell(x)$ at small x and this is given in Shankar.)

Also, there is an error in the formula on p. 29 of the notes, where I relate β_ℓ to the phase shift. The correct formulae are

$$\sin \delta_\ell = \frac{\beta_\ell}{\sqrt{1 + \beta_\ell^2}} \quad (2)$$

and

$$\cos \delta_\ell = -\frac{1}{\sqrt{1 + \beta_\ell^2}}. \quad (3)$$

This error stems from another error at the bottom of p. 28. On the very last line, the quantity $|K_\ell|$ should be replaced by $|K_\ell|/i$.

With this information, I believe the last problem of this set becomes doable in finite time.