(20 pts.) Consider scattering by a repulsive delta-shell potential defined by:

\[
\frac{2mV(r)}{\hbar^2} = \gamma \delta(r - R),
\]

(1)

with \( \gamma > 0 \). (a). Set up an equation that determines the s-wave phase shift, \( \delta_0 \), as a function of \( k \). \([E = \hbar^2 k^2 / (2m)].\) (b). Assume now that \( \gamma \) is very large,

\[
\gamma > \frac{1}{R}; \gamma \gg k.
\]

(2)

Show that if \( \tan(kR) \) is not close to zero, the s-wave phase shift resembles the hard-sphere result discussed in class and in the text. Show also that for \( \tan(kR) \) close to (but not exactly equal to) zero, resonance behavior is possible; that is \( \cot \delta_0 \) goes through zero from the positive side as \( k \) increases. Determine approximately the positions of the resonances keeping terms of order \( 1/\gamma \); compare these with the bound-state energies for a particle confined inside an infinitely hard spherical well of the same radius,

\[
V(r) = 0; r < R; \quad V(r) = \infty, \quad r > R.
\]

(3)

Also, obtain an approximate expression for the resonance width \( \Gamma \) defined by

\[
\Gamma = -\frac{2}{[d(\cot \delta_0)/dE]_{E=Ec}}.
\]

(4)

Show that these resonances become extremely sharp as \( \gamma \) becomes large.