Physics 829: Problem Set 3

Due Wednesday, April 16, 2008 at 11:59 PM

Each problem is worth 10 points unless otherwise specified

(20 pts.) Consider scattering by a repulsive delta-shell potential defined by:

$$\frac{2mV(r)}{\hbar^2} = \gamma\delta(r-R), \tag{1}$$

with $\gamma > 0$. (a). Set up an equation that determines the s-wave phase shift, δ_0, as a function of k. [$E = \hbar^2k^2/(2m)$.] (b).Assume now that γ is very large,

$$\gamma \gg \frac{1}{R}; \gamma \gg k. \tag{2}$$

Show that if $\tan(kR)$ is *not* close to zero, the s-wave phase shift resembles the hard-sphere result discussed in class and in the text. Show also that for $\tan(kR)$ close to (but not exactly equal to) zero, resonance behavior is possible; that is $\cot\delta_0$ goes through zero from the positive side as k increases. Determine approximately the positions of the resonances keeping terms of order $1/\gamma$; compare these with the bound-state energies for a particle confined *inside* an infinitely hard spherical well of the same radius,

$$V(r) = 0, \quad r < R; \quad V(r) = \infty, \quad r > R. \tag{3}$$

Also, obtain an approximate expression for the resonance width Γ defined by

$$\Gamma = -\frac{2}{[d(\cot\delta_0)/dE]_{E=E_r}}. \tag{4}$$

Show that these resonances become extremely sharp as γ becomes large.