**Physics 828: Problem Set VII**

**Dr. Stroud**

**Due Wednesday, March 4 by 11:59 P. M.**

Each problem is worth 10 pts. unless otherwise specified.

1. Shankar, problem 18.2.2.
2. Shankar, problem 18.2.6.
3. **Density of free particle states in one dimension.** In class, I showed that for a free particle of mass $m$, the density of states in energy $\rho(E) = AE^{1/2}$, where $E$ is the energy, and $A$ is a constant. We also derived the value of the constant. If the particle is spinless, we found that $A = V(2m/\hbar^2)^{3/2}/(4\pi^2)$, where $V$ is the volume of the system, and $\rho(E)$ is defined by the statement that the number of free particle states between $E$ and $E + dE$ is $\rho(E)dE$.

   Now carry out the same calculation for a free particle of mass $m$ in 1D. Again, as in class, use periodic boundary conditions to find the allowed values of $k$ in a system of length $L$ along the x axis. Show that the density of states in this case, assuming spinless particles, is

   $$\rho(E) = \frac{L}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E^{-1/2}. \tag{1}$$

   Note that this expression includes states of both positive and negative k of the same energy.

4. **Simple model of the photoelectric effect in one dimension.** Consider,in a one-dimensional problem, a particle of mass $m$ placed in a potential of the form $V(x) = -\alpha\delta(x)$, where $\alpha$ is a real positive constant. Recall that in such a potential there is a single bound state, with energy $E_0 = -m\alpha^2/(2\hbar^2)$, corresponding to a normalized wave function $\psi_0(x) = \sqrt{m\alpha/\hbar^2}\exp(-m\alpha|x|/\hbar^2)$.

   The particle (assumed to be of charge q) interacts with an electric field oscillating with angular frequency $\omega$. The corresponding perturbation is $W(t) = -q\mathcal{E}X\sin(\omega t) \equiv W\sin(\omega t)$, where $\mathcal{E}$ is a constant.

   (a) Calculate the matrix element $\langle k|X|\psi_0\rangle$ of the position observable X between the bound state $|\psi_0\rangle$ and the positive energy state $|\mathbf{k}\rangle$ whose normalized wave function is approximated as a free particle state $(1/L^{1/2})\exp(ikx)$.

   (b). Hence, calculate, using the Fermi Golden Rule, the transition probability $w$ per unit time to an arbitrary positive energy state. How does $w$ vary with $\omega$ and with the final energy $E_f$ of the charged particle?

   The Fermi Golden Rule, for this case, reads

$$w = \frac{\pi}{2\hbar}|k_f|W|\psi_0\rangle|^2\rho(E_f = E_0 + \hbar\omega). \quad (2)$$

   This differs by a factor of (1/4) from that given in class, because the time-dependence of the perturbation is $sin(\omega t)$ rather than $\exp(-i\omega t)$.

   Comment: the final states, the density of final states, and the matrix element can actually all be calculated exactly.