Physics 828: Problem Set 4

Dr. Stroud

Due Wednesday, February 4 at 11:59 P. M.

1. Shankar, problem 16.1.1.
2. Shankar, problem 16.1.3.
3. Shankar, problem 16.2.4.
4. Shankar, problem 16.2.7.
5. (20 pts.) **Jaynes-Cummings model.** The Jaynes-Cummings model is a Hamiltonian, originally introduced in quantum optics, which has found wide application in many areas of physics. It deals with a spin-1/2 particle coupled to a harmonic oscillator of frequency ω. The Hamiltonian is

$$H_{JC} = \hbar\omega a^\dagger a + \frac{\hbar\Omega}{2}\sigma_z + \hbar g(\sigma^+ a + \sigma^- a^\dagger). \tag{1}$$

Here $a^\dagger$ and a are the raising and lowering operators for the harmonic oscillator, and σ^+, σ^- and σ_z are the spin-1/2 operators discussed in the book and in class.

(a). Show that H_{JC} commutes with the operator $a^\dagger a + \frac{1}{2}\sigma_z$. Thus, $a^\dagger a + \frac{1}{2}\sigma_z$ is a constant of the motion.

(b). A suitable orthonormal basis consists of the states $|\uparrow n\rangle$ and $|\downarrow n$, where $\uparrow$ and $\downarrow$ denote the up and down spin states, and n is the harmonic oscillator quantum number.

Explain why, when $\omega = \Omega$, the states $|\uparrow n\rangle$ and $|\downarrow n{+}1\rangle$ are degenerate if g = 0. Show that, when $g \neq 0$, the degeneracy is broken, and, using the result of part (a), calculate the splitting between the formerly degenerate states in this case.