

Additional Problem for Prob. Set 2

(5 pts.) In class, I mentioned that the interaction energy between a charge q and a weak magnetic field $\mathbf{B} = B\hat{z}$ is

$$H_{int} = -\frac{qBL_z}{2mc}, \quad (1)$$

where m is the mass of the charged particle and L_z is an orbital angular momentum operator. In this problem, you will derive this result.

It was shown last quarter that the Hamiltonian of a charged particle interacting with a vector potential \mathbf{A} is

$$H = \frac{1}{2m}(\mathbf{p} - \frac{q}{c}\mathbf{A})^2. \quad (2)$$

By expanding this Hamiltonian to first order in \mathbf{A} and using the gauge

$$\mathbf{A} = \frac{B}{2}(x\hat{y} - y\hat{x}), \quad (3)$$

show that the interaction energy, to first order in B , takes the form (1).