Physics 828: Problem Set 4

Dr. Stroud

Due Friday, February 8 at 11:59 P. M.

NOTE NEW DUE DATE.


5. (20 pts.) **Jaynes-Cummings model.** NOTE: THIS PROBLEM IS POSTPONED TO PROB. SET 5. The Jaynes-Cummings model is a Hamiltonian, originally introduced in quantum optics, which has found wide application in many areas of physics. It deals with a spin-1/2 particle coupled to a harmonic oscillator of frequency $\omega$. The Hamiltonian is

$$H_{JC} = \hbar \omega a^\dagger a + \frac{\hbar \Omega}{2} \sigma_z + \hbar g (\sigma^+ a + \sigma^- a^\dagger).$$  

Here $a^\dagger$ and $a$ are the raising and lowering operators for the harmonic oscillator, and $\sigma^+$, $\sigma^-$ and $\sigma_z$ are the spin-1/2 operators discussed in the book and in class.

(a). Show that $H_{JC}$ commutes with the operator $a^\dagger a + \frac{1}{2} \sigma_z$. Thus, $a^\dagger a + \frac{1}{2} \sigma_z$ is a constant of the motion.

(b). A suitable orthonormal basis consists of the states $| \uparrow n \rangle$ and $| \downarrow n \rangle$, where $\uparrow$ and $\downarrow$ denote the up and down spin states, and $n$ is the harmonic oscillator quantum number.

Explain why, when $\omega = \Omega$, the states $| \uparrow n \rangle$ and $| \downarrow n+1 \rangle$ are degenerate if $g = 0$. Show that, when $g \neq 0$, the degeneracy is broken, and, using the result of part (a), calculate the splitting between the formerly degenerate states in this case.