

Tilt modulus and angle-dependent flux lattice melting in the lowest Landau level approximation

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For a clean high- T_c superconductor, we analyze the Lawrence-Doniach free energy in a tilted magnetic field within the lowest Landau level approximation. The free energy maps onto that of a strictly c -axis field, but with a reduced interlayer coupling. We use this result to calculate the tilt modulus C_{44} of a vortex lattice and vortex liquid. The vortex contribution to C_{44} can be expressed in terms of the squared c -axis Josephson plasmon frequency ω_{pl}^2 . The transverse component of the field has very little effect on the position of the melting curve.

I. INTRODUCTION

This paper is concerned with the tilt modulus C_{44} of the vortex system in a high- T_c or otherwise layered superconductor. C_{44} is an elastic constant which measures the free energy cost of applying a small transverse field in addition to a field applied parallel to the c axis. It is a relevant parameter in many physical processes, such as collective pinning,¹ the ‘‘peak effect,’’^{1,2} and the transition to the Bose glass state.³ Previous investigations have shown that C_{44} is very strongly dependent on the wave vector \mathbf{k} of the transverse field.^{4,5} In addition, it is finite in the flux liquid as well as the flux solid phase and is strongly affected by different kinds of disorder.^{1,6,7}

A number of authors have analyzed C_{44} in various approximations. Several groups^{4,5,8} have calculated $C_{44}(\mathbf{k})$ in a flux lattice as a function of the wave vector \mathbf{k} of the transverse magnetic field. Other authors have considered C_{44} in the flux liquid state,^{7,9} and in the presence of disorder in both the solid and liquid phases.^{6,7}

In this paper, we calculate C_{44} for a layered superconductor at *high fields*. We describe the superconductor using a Lawrence-Doniach¹⁰ free energy functional, and we evaluate fluctuations in the lowest Landau level (LLL) approximation, appropriate for strong c -axis magnetic fields.¹¹ This LLL approach accounts well for the position of the flux lattice melting curve in the magnetic-field/temperature plane,^{11,12} as well as the value of the magnetization in both solid and liquid phases,¹³ both in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ and the less anisotropic $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$. To obtain C_{44} , we generalize the LLL approximation to apply to fields with a finite ab component. For a defect-free system in the LLL approximation, the tilted field free energy can be exactly mapped onto the usual free energy for a field $\mathbf{B}||c$, but with a weaker interlayer coupling which depends on tilt angle.

As a byproduct of this transformation, we can also analyze the behavior of the flux lattice melting curve in a magnetic field tilted at an angle to the c axis. We find that this curve is little affected by the presence of a transverse magnetic field component at fields where the LLL approximation is applicable, consistent with available (but limited) experiment.

II. LOWEST LANDAU LEVEL APPROXIMATION WITH TILTED FIELD

At fixed external magnetic field $\mathbf{H}=H_z\hat{\mathbf{z}}+H_x\hat{\mathbf{x}}$, the Lawrence-Doniach Gibbs free energy in a form which incorporates the field energy is¹⁰

$$\begin{aligned}
 G[\psi, \mathbf{A}] &= d \sum_n \int d^2\mathbf{r} \left\{ a(T) |\psi_n(\mathbf{r})|^2 \right. \\
 &\quad \left. + t |\psi_n(\mathbf{r}) e^{-i(2\pi/\Phi_0)A_z d} - \psi_{n-1}(\mathbf{r})|^2 \right. \\
 &\quad \left. + \frac{1}{2m_{ab}} \left| \left(-i\hbar\nabla_{\perp} - \frac{q}{c}\mathbf{A}_{\perp} \right) \psi_n(\mathbf{r}) \right|^2 + \frac{b}{2} |\psi_n(\mathbf{r})|^4 \right\} \\
 &\quad + V \frac{|\mathbf{B} - \mathbf{H}|^2}{8\pi} \\
 &\equiv \tilde{G}[\psi, \mathbf{A}] + V \frac{|\mathbf{B} - \mathbf{H}|^2}{8\pi}. \tag{1}
 \end{aligned}$$

Here ψ_n is the order parameter in the n^{th} layer, d is the distance between layers, $q = -2|e|$, and $t = \hbar^2/(2m_c d^2)$ is the interlayer coupling energy. In a cuprate superconductor such as $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, d represents the repeat distance in the c direction. ∇_{\perp} is the xy component of the gradient operator, V is the system volume, and the magnetic induction $\mathbf{B} = \nabla \times \mathbf{A}$, where the vector potential $\mathbf{A} = (A_x, A_z)$.

We choose the gauge $\mathbf{A} = -B_z y \hat{\mathbf{x}} + B_x y \hat{\mathbf{z}}$ and expand the ψ_n 's as a linear combination of LLL states, to obtain $\psi_n(\mathbf{r}) = (\sqrt{3}a_H^2(T)/4b^2)^{1/4} \sum_{k,c_{k,n}} e^{ikx - (y - kl^2)^2/2l^2}$, where $k = 2\pi p/L_x$ is the quantized momentum (p a positive integer), $l = \sqrt{\Phi_0/2\pi B_z}$ is the magnetic length, $a_H(T) = [a(T) + 2t][1 - B_z/H_{c2}(T)]$, and sample volume is $V = L_x L_y L_z$. In terms of measurable quantities, $a(T) + 2t = -\hbar q H_{c2}(T)/(2m_{ab}c)$, where m_{ab} is the transverse effective mass and $H_{c2}(T)$ is the upper critical field as a function of temperature T . Substituting this expansion into Eq. (1) and integrating over x and y yields

$$\begin{aligned} \tilde{G} = & \frac{a_H^2(T)\pi l^2 d}{b} N_x \sum_{k,n} \left\{ -|c_{k,n}|^2 - \frac{t \exp(-(\pi B_x dl/\Phi_0)^2)}{|a_H(T)|} \right. \\ & \times (c_{k,n}^* c_{k,n-1} e^{-i2\pi B_x dl^2 k/\Phi_0} + c.c.) \\ & \left. + \frac{3^{1/4}}{5^{5/2}} \sum_{q_1, q_2} c_{k,n} c_{k+q_1, n}^* c_{k+q_2, n}^* c_{k+q_1+q_2, n} e^{-(q_1^2+q_2^2)l^2/2} \right\}, \end{aligned} \quad (2)$$

where $N_x = (L_x/l)(\sqrt{3}/4\pi)^{1/2}$. This expresses the Gibbs free energy as a function of the expansion coefficients $c_{k,n}$ for a tilted magnetic field. This expression differs from that for $\mathbf{B} \parallel \hat{\mathbf{z}}$ in only two respects:¹¹ (i) there is an extra phase factor in the interlayer coupling; and (ii) the strength of the interlayer coupling is renormalized by an exponentially decaying factor.

The phase factor in Eq. (2) can be eliminated by introducing a new set of coefficients $\tilde{c}_{k,n} \equiv c_{k,n} e^{-i(2\pi B_x dl^2 k/\Phi_0)n}$, in Eq. (2). The product term becomes $c_{k,n}^* c_{k,n-1} e^{-i2\pi B_x dl^2 k/\Phi_0} = \tilde{c}_{k,n}^* \tilde{c}_{k,n-1}$. This transformation does not affect the free energy term which contains products of four coefficients. To see this, note that when such a typical product is transformed, it picks up a phase factor $e^{i(2\pi B_x dl^2 n/\Phi_0)(k-(k+q_1)-(k+q_2)+(k+q_1+q_2))}$, where each $(k+q_i)$ term is an integer modulo N_Φ , N_Φ being the number of flux lines in the sample. The terms in the exponential can be summed to yield zero, leaving this term unaltered. Similarly, $|c_{k,n}|^2 = |\tilde{c}_{k,n}|^2$.

In terms of the new variables, the free energy is thus

$$\begin{aligned} \tilde{G} = & \frac{a_H^2(T)\pi l^2 d N_x}{b} \sum_{k,n} \left\{ -|\tilde{c}_{k,n}|^2 - \frac{t'}{|a_H(T)|} \right. \\ & \times (\tilde{c}_{k,n}^* \tilde{c}_{k,n-1} + c.c.) + \frac{3^{1/4}}{5^{5/2}} \\ & \left. \times \sum_{q_1, q_2} \tilde{c}_{k,n} \tilde{c}_{k+q_1, n}^* \tilde{c}_{k+q_2, n}^* \tilde{c}_{k+q_1+q_2, n} e^{-(q_1^2+q_2^2)l^2/2} \right\}, \end{aligned} \quad (3)$$

where

$$t' = t e^{-(\pi B_x dl/\Phi_0)^2} \quad (4)$$

is the renormalized interlayer coupling. Thus, the tilted B -field free energy is *identical in form* to the free energy for a field $\mathbf{B} \parallel \hat{\mathbf{z}}$, but with a *weaker* interlayer coupling. The ground state solution for the redefined amplitudes $\tilde{c}_{k,n}$ are identical to those for the $c_{k,n}$'s in a strictly longitudinal field but weaker $\mathbf{B} \parallel \hat{\mathbf{z}}$. However, the order parameter ψ_n picks up a B_x dependency,

$$\begin{aligned} \psi_n(\mathbf{r}) = & \left(\frac{\sqrt{3} a_H^2(T)}{4b^2} \right)^{1/4} \\ & \times \sum_k \tilde{c}_{k,n} e^{-ik(2\pi B_x dl^2/\Phi_0)n} e^{ikx - (y-kl)^2/2l^2}, \end{aligned} \quad (5)$$

corresponding, at low temperatures, to a tilted Abrikosov lattice.

III. TILT MODULUS

C_{44} is defined by

$$C_{44} = \frac{1}{V} \left(\frac{\partial^2 \mathcal{G}}{\partial \theta^2} \right)_{\theta=0}, \quad (6)$$

where θ is the angle between \mathbf{H} and the c axis, and where \mathcal{G} , the Gibbs free energy appropriate to an experiment at constant \mathbf{H} and T , is given by

$$\begin{aligned} \mathcal{G} = & -k_B T \ln \int \prod_{k,n} d\tilde{c}_{k,n}^* d\tilde{c}_{k,n} e^{-\tilde{G}/k_B T + V \frac{|\mathbf{B}-\mathbf{H}|^2}{8\pi}} \\ = & \mathcal{G}^v + V \frac{|\mathbf{B}-\mathbf{H}|^2}{8\pi}. \end{aligned} \quad (7)$$

Then, writing $\partial/\partial\theta \rightarrow H_z \partial/\partial H_x$ for small H_x , we find that the second term in Eq. (7) contributes a term $C_{44}^c = H_z^2/(4\pi) \sim B_z^2/(4\pi)$ in the LLL regime, where the magnetization is small. This is the compressive contribution to the tilt modulus.

To calculate the remaining (vortex-related) contribution to C_{44} , which we denote C_{44}^v , we first write $\tilde{G}/k_B T = \mathcal{H}/T$, where $T = b k_B T / a_H^2(T) \pi l^2 d$ and

$$\begin{aligned} \mathcal{H} = & N_x \sum_{k,n} \left\{ -|c_{k,n}|^2 - \frac{t \exp(-(\pi B_x dl/\Phi_0)^2)}{|a_H(T)|} \right. \\ & \times (\tilde{c}_{k,n}^* \tilde{c}_{k,n-1} + c.c.) \\ & \left. + \frac{3^{1/4}}{2^{5/2}} \sum_{q_1, q_2} \tilde{c}_{k,n} \tilde{c}_{k+q_1, n}^* \tilde{c}_{k+q_2, n}^* \tilde{c}_{k+q_1+q_2, n} e^{-(q_1^2+q_2^2)l^2/2} \right\}. \end{aligned} \quad (8)$$

Then, writing $\partial^2/\partial\theta^2 = B_z^2 \partial^2/\partial B_x^2$, taking $B_z \sim H_z$, and evaluating the derivatives from Eqs. (7), with the result

$$\begin{aligned} C_{44}^v = & \left(\frac{\sqrt{3}}{8} \right)^{1/2} \left(\frac{\pi t d^3 |a(T) + 2t|}{\sqrt{\Phi_0} b L_y L_z} \right) (1 - B_z/H_{c2}) \\ & \times \sqrt{B_z} \left\langle \sum_{k,n} (c_{k,n}^* c_{k,n-1} + c.c.) \right\rangle T, \end{aligned} \quad (9)$$

where we have dropped the tildes on the $c_{k,n}$'s because this expression is evaluated at $B_x = 0$. Equation (9) can be expressed in terms of measurable quantities using the identities $a(T) + 2t = \hbar q H_{c2}(T)/2m_{ab}c$ and $b = 2\pi\kappa^2(q\hbar/m_{ab}c)^2$, where $\kappa = \lambda_{ab}(B_z, T)/\xi_{ab}(B_z, T)$ is the ratio of the ab penetration depth and coherence length. We also write $t = \hbar^2/(2m_{ab}d^2\gamma^2)$, where γ^2 is the anisotropy parameter. The result is

$$C_{44}^v = \frac{1}{32\pi\kappa^2\gamma^2} [H_{c2}(T) - B_z] B_z \frac{N_x}{N_\Phi N_z} \times \left\langle \sum_{k,n} c_{k,n}^* c_{k,n-1} + c.c. \right\rangle T. \quad (10)$$

Finally, the total tilt modulus is

$$C_{44} \sim \frac{B_z^2}{4\pi} + C_{44}^v. \quad (11)$$

In a triangular Abrikosov lattice, for example, only $N_y = N_\phi/N_x$ of the N_ϕ coefficients in each layer are nonzero. The nonzero coefficients are $c_{2pN_x, n} = (2/\beta_A)^{1/2}$, $c_{(2p+1)N_x, n} = i(2/\beta_A)^{1/2}$, where p is an integer ranging from 0 to N_y and $\beta_A = 1.159595 \dots$ is the Abrikosov ratio.¹⁴ Substituting these values into Eq. (10), using $N_y/L_y = 2N_x/(L_x\sqrt{3})$, and adding the compressive term, we obtain

$$C_{44} = \frac{B_z^2}{4\pi} + \frac{1}{8\pi\beta_A\kappa^2\gamma^2} (H_{c2}(T) - B_z) B_z. \quad (12)$$

Equation (12) can be compared to a previous estimate^{9,15} $C_{44}^{LVG} = (B_z^2/4\pi)[1 + 1/(4\pi\tilde{\lambda}_{ab}^2(B_z, T)\gamma^2 n_s)]$, where n_s is the superfluid number density for a two-dimensional Bose system related to the three-dimensional flux line system by a path integral formalism.³ Writing $\lambda_{ab}(B_z, T) = \kappa\xi_{ab}(B_z, T)$, $1/\xi_{ab}^2(B_z, T) \sim 2\pi(H_{c2}(T) - B_z)/\Phi_0$, and approximating n_s by $n_0 = B_z/\Phi_0$, the number of bosons (i.e., flux lines) per unit area,^{6,7} we obtain $C_{44}^{LVG} \sim B_z^2/(4\pi) + [(H_{c2}(T) - B_z)B_z]/(8\pi\kappa^2\gamma^2)$. This result is in agreement with our own result, Eq. (12), except for the factor $(1/\beta_A) \sim 0.86$, which arises from the nonuniformity of n_s in the Abrikosov flux phase.

Our C_{44}^v is closely connected to the so-called Josephson plasmon frequency ω_{pl} , as calculated in the same LLL approximation.¹⁶ $\omega_{pl}^2(B_z, T)$ is the squared plasma frequency of the Josephson junction formed between adjacent ab layers, and is given by¹⁶

$$\omega_{pl}^2(B, T) = (\omega_{pl}^{MF})^2 \frac{\beta_A}{2} [\beta_A N_x / (2N_\Phi N_z)] \times \left\langle \sum_{k,n} (c_{k,n}^* c_{k,n-1} + c.c.) \right\rangle T, \quad (13)$$

where $\omega_{pl}^{MF} = \sqrt{[(H_{c2}(T) - B)cq]/[\epsilon_0\kappa^2\gamma^2\hbar\beta_A]}$ is the mean-field Josephson plasmon frequency and ϵ_0 is an inter-layer dielectric constant. Combining Eqs. (10) and (13) gives

$$\frac{C_{44}^v(B_z, T)}{\epsilon_0\omega_{pl}^2(B_z, T)} = \frac{B_z\hbar^2}{8\Phi_0q^2}. \quad (14)$$

Figure 1 shows $C_{44}^v(B_z, T)$ for $B_z = 2$ T in clean $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, using this relation ω_{pl}^2 as calculated in Ref. 16. Like ω_{pl}^2 , C_{44}^v has a discontinuity at the flux lattice melting transition, and remains finite in the flux liquid state. Experiments on ω_{pl}^2 are consistent with this result, indicating

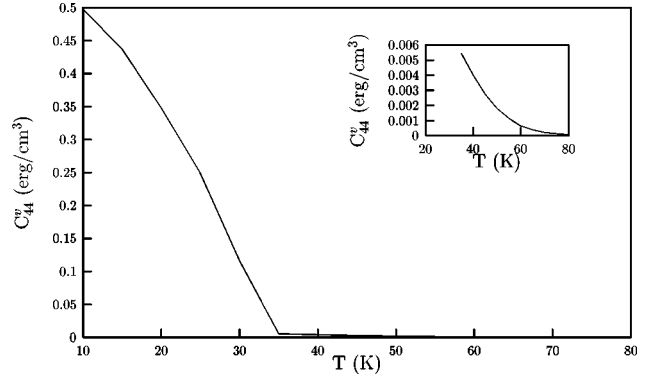


FIG. 1. Vortex contribution C_{44}^v to the tilt modulus, plotted versus temperature for $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ at $B_z = 2$ T, as evaluated using Eq. (14) and ω_{pl}^2 from Ref. 16. C_{44}^v is discontinuous at the flux lattice melting temperature $T_M(B_z)$ (indicated by arrow). Inset: enlargement of C_{44}^v near and above T_M .

that ω_{pl}^2 , and hence C_{44}^v , have discontinuities at flux lattice melting in clean $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$.^{17,18}

IV. FLUX LATTICE MELTING IN A TILTED MAGNETIC FIELD

Equations (3) and (4) have some striking implications for the LLL phase diagram of a clean layered superconductor. When $\mathbf{B} \parallel c$, this phase diagram depends on only two parameters, namely $g = a_H \sqrt{\pi} l^2 d / (b k_B T)$ and $\eta = t / |a_H|$.¹² Our results show that, even with a transverse field B_x , the phase diagram still depends on only *two* parameters, except that η is now replaced by $\eta' = \eta \exp[-(\pi B_x^2 d^2)/(2B_z \Phi_0)]$. As found previously,¹¹⁻¹³ this phase diagram contains a single first-order melting line separating a triangular vortex lattice from a vortex liquid. In the regime where the LLL approximation is adequate, our results show that this first-order line should persist in a tilted magnetic field (cf. Fig. 2). Furthermore, in most high- T_c materials, the line is shifted very little by a

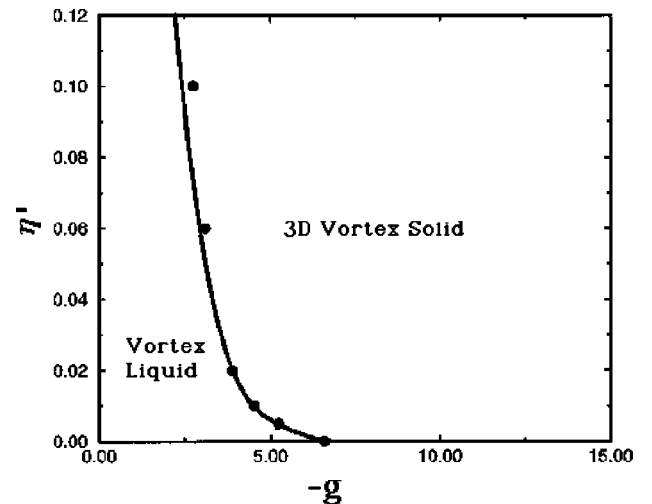


FIG. 2. Phase diagram of a clean high- T_c material in the LLL approximation, including a transverse magnetic field component B_x . The parameters η' and g are defined in the text. Points and spline fit are given by Hu and MacDonald (Ref. 12).

nonzero B_x . (For $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, $t' \sim t \exp[-7.5 \times 10^{-7} B_x^2/B_z]$.)

We have found no experimental melting data in tilted magnetic fields at fields where the LLL approximation is applicable. In $\text{BiSr}_2\text{Ca}_2\text{Cu}_2\text{O}_{8+x}$, the low-field melting temperature has been reported¹⁹ to depend only on the c component of \mathbf{H} , which, though obtained in a very different regime, would be consistent with the result of our calculations. It would be of great interest to have further tests of the LLL predictions in the relevant high-field regime.

V. SUMMARY

In this paper, we have extended the LLL approximation for high- T_c superconductors to treat fields tilted at an angle to the layer perpendicular. The resulting free energy has exactly the same *form* as the usual case, except that the effective interlayer coupling is reduced. For high- T_c materials, this reduction is small; hence, we predict that the flux lattice

melting temperature will be little affected by the application of an oblique magnetic field in the range where the LLL approximation is valid, except possibly at angles nearly parallel to the layers. This prediction appears consistent with some existing (but low-field) experiments.¹⁹ We also obtain an expression for the zero-wave-vector tilt modulus C_{44} , in good agreement with previous estimates by other means.^{9,15} Finally, the vortex contribution C_{44}^v is proportional to the squared Josephson plasmon frequency ω_{pl}^2 , as calculated in the same LLL approximation, and remains finite in the vortex liquid as well as the vortex solid phase.

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