Physics 880K20: How to Use Kerr Medium to Generate a Two-Qubit Operation

In my lecture of February 6, I misstated the correct way to use a Kerr medium to generate a two-qubit operation. The correct way is described below.

As stated in class, a single qubit requires *two cavities*, but always has exactly one photon in the two cavities. We may label the basis of the single qubit state as $|10\rangle$ and $|01\rangle$, where the first number is the number of photons in the first cavity, and the second to the number in the second cavity. Thus, a two-qubit state would involve *four* cavities. We can denote the four possible states of the two qubits as $|1010\rangle$, $|1001\rangle$; $|0110\rangle$, and $|0101\rangle$. The four entries refer to the four cavities needed for the two qubits. We label these cavities a, b, c, and d. Note that there are always exactly two photons in the two qubits.

Now, to produce the proper two-qubit operation, we insert a Kerr medium so that any photon in cavities b and d will go through the Kerr medium. The phase shift generated by the Kerr medium is $K = \exp[i\chi b^\dagger b d^\dagger d n L/(\hbar c)]$. Let us assume that the length of the Kerr medium is such as to generate a π phase shift if $b^\dagger b = d^\dagger d = 1$, i. e. $\exp[i\chi L/(\hbar c)] = \exp(i\pi) = -1$. Then the effect of the Kerr medium on the four states can be summarized as follows:

$$\begin{aligned} K|1010\rangle &= |1010\rangle; \\ K|1001\rangle &= |1001\rangle \\ K|0110\rangle &= |0110\rangle \\ K|0101\rangle &= -|0101\rangle. \end{aligned} \tag{1}$$

Now we change the labeling so that instead of referring to states of the four cavities, we refer to states of the two qubits. Thus, we denote the state $|1010\rangle$ as $|00\rangle_Q$, where the subscript Q denotes that this is a two qubit state. Similarly we write $|1001\rangle = |01\rangle_Q$; $|0110\rangle = |10\rangle_Q$; and $|0101\rangle = |11\rangle_Q$. Then the previous equation, expressed in terms of the qubits, takes the form

$$\begin{aligned} K|00\rangle_Q &= |00\rangle_Q; \\ K|10\rangle_Q &= |10\rangle_Q; \\ K|01\rangle_Q &= |01\rangle_Q; \\ K|11\rangle_Q &= |11\rangle_Q. \end{aligned} \tag{2}$$

In other words, the operator K for a Kerr medium, arranged in the geometry just described and with the basis just described, has non-zero matrix elements $K_{11} = K_{22} = K_{33} = 1$; $K_{44} = -1$, with all off-diagonal elements zero.