I. INTRODUCTION

The most anisotropic high-$T_c$ superconductors behave rather like a collection of underdamped Josephson junctions stacked parallel to the c axis. Recent microwave experiments in these materials see clear evidence of such junctions, in the form of Josephson plasmon oscillations. For example, in $c$-axis polarized reflectivity measurements a strong absorption is measured at a frequency which is interpreted as the Josephson plasmon frequency, $\omega_{pl}(B,T)$, where $B$ is the magnetic induction parallel to $c$ and $T$ the temperature. In single crystals of $La_{2-x}Sr_xCuO_4$, these experiments yield $\omega_{pl}\sim 20-50 \text{ cm}^{-1}$, i.e., $600-1500 \text{ GHz}$, but in the more anisotropic $Bi_2Sr_2Ca_2Cu_3O_{8+\delta}$ they give $\omega_{pl}\sim 30-80 \text{ GHz}$,$^2$ much smaller than the superconducting gap and weakly damped. On the theory side, Tachiki et al.$^4$ have calculated the $c$-axis plasmon-induced reflectivity and transmissivity at finite $B$ in $Bi_2Sr_2Ca_2Cu_3O_{8+\delta}$. Bulaevskii and co-workers$^5$ have derived a relation between $\omega_{pl}$ and the interlayer Josephson coupling energy, while Koshelev has obtained the leading terms in a high-$T$ expansion for $\omega_{pl}(B,T).^6$

In this paper, we calculate $\omega_{pl}(B,T)$ within the Lawrence-Doniach (LD) model, incorporating thermal fluctuations within the lowest Landau level (LLL) approximation. This model, which is suitable at high $B$ and $T$, accounts well for first-order flux lattice melting in clean samples of both $Bi_2Sr_2Ca_2Cu_3O_{8+\delta}$ and $YBa_2Cu_3O_7.7-9$ We calculate $\omega_{pl}(B,T)$ by combining this model with a simple theory for the electrodynamics of high-$T_c$ superconductors, based on Maxwell’s equations. In some respects, but not all, our results agree qualitatively with measurements of $\omega_{pl}$ over a range of $B$ and $T$. In particular, we find that $\omega_{pl}$ remains nonzero even in the flux liquid. In contrast to experiment, however, the calculated $\omega_{pl}$ falls discontinuously at melting into the flux liquid state, and the contours of constant $\omega_{pl}$ in the $B$-$T$ plane do not cross the flux lattice melting curve. When we extend the model to include columnar disorder, we find that, in this case, $\omega_{pl}(B,T)$ is continuous in $T$ across the irreversibility line, in agreement with experiment, though the constant $\omega_{pl}$ contours still remain parallel to the melting transition line (which in this case is generally considered to be a glass transition). Finally, we obtain a formally exact expression, valid beyond the LLL model and in disordered as well as ordered samples, for $\omega_{pl}$ in the flux liquid state.

II. MODEL

In the LD model, the complex superconducting order parameter $\psi_n(\mathbf{r})$ at a point $\mathbf{r}= (x,y)$ within the $n$th CuO$_2$ layer is described by the Ginzburg-Landau free energy functional $F[\psi] = \int d\mathbf{r} \Sigma_n F_n[\psi]$, where

$$F_n[\psi] = a(T) |\psi_n(\mathbf{r})|^2 + \frac{b}{2} |\psi_n(\mathbf{r})|^4 + \frac{1}{2m_{ab}} \left( -i\hbar \nabla - \frac{e^*}{c} \right)^2 \phi_n(\mathbf{r})^2 + t \left| \psi_{n+1}(\mathbf{r}) - \psi_n(\mathbf{r}) \right| \exp \left( \frac{2\pi i \gamma (n+1)}{\Phi_0} \right) A_n d\mathbf{z} \right|^2. \tag{1}$$

Here $d$ is the superconducting layer thickness, $s$ is the repeat distance between layers, $e^* = 2e$, $\Phi_0 = \hbar c/e^*$, $A_n$ and $\Lambda_n$ are components of the vector potential, and $a(T)$, $b$, $m_{ab}$, and $m_c$ are material-dependent parameters. The interlayer Josephson coherence $t = \hbar^2/(2m_s s^2) = \hbar^2/(2m_{ab}s^2 \gamma^2)$, where $\gamma = \sqrt{m_c/m_{ab}}$. It is readily shown that the free energy (1) is invariant under the gauge transformation $A \rightarrow A + \nabla \Lambda$, $\psi_n(\mathbf{x}) \rightarrow \psi_n(\mathbf{x}) \exp[ie^* \Lambda(\hbar c)]$, where $\Lambda(\mathbf{x})$ is an arbitrary function of $\mathbf{x}= (r,z)$. Thus, the free energy is applicable, in principle, to magnetic fields with components perpendicular as well as parallel to the $c$ axis.

In the extreme-type-II high-$T_c$ materials, the magnetic induction $\mathbf{B}$ and field $\mathbf{H}$ are approximately equal. Maxwell’s equations for the $z$ component of the electric field $\mathbf{E}$ then give

$$\nabla^2 E_z = 4\pi \left[ \frac{1}{\varepsilon_0} \frac{\partial^2}{c^2 \partial t^2} + \frac{1}{c} \frac{\partial}{\partial t} J_z \right]. \tag{2}$$
where $\rho$ is the charge density, $J_z$ is the $z$ component of current density, and $\epsilon_0$ is an appropriate dielectric constant. We seek a wave-like solution to this differential equation, such that $\nabla \cdot \rho = 0$. We also include in $J_z$ only the supercurrent density, and neglect the normal current in parallel, which damps the plasmon. The supercurrent density in the region $ns\leq z\leq (n+1)s$ can then be written

$$\vec{J}_c(x,t) = J_{n,n+1}(r) \sin \theta_{n,n+1}(r),$$

(3)

where $\theta_{n,n+1}(r)$ is the gauge-invariant phase difference between layers $n$ and $n+1$ at position $r$ and time $t$, and $J_{n,n+1}(r)$ is the critical current density between those layers. We also use the Josephson relation

$$\frac{\partial \theta_{n,n+1}(r,t)}{\partial t} = \frac{e^* s}{\hbar} (E_z(r,t))_{n,n+1},$$

(4)

where $\langle \cdots \rangle_{n,n+1}$ denotes the average of the quantity in brackets along the $z$ axis in the region $ns\leq z\leq (n+1)s$. Following the treatment of Ref. 5, we now assume that $E_z$ is slowly varying along the $z$ direction, and replace $\langle E_z(r,t) \rangle_{n,n+1}$ by $E_z(r,z,t)$; we also express the $\theta_{n,n+1}(r,t)$ as the sum of a time-independent part $\theta_{n,n+1}^0(r)$ and a small time-dependent part $\delta \theta_{n,n+1}(r,t)$. Then differentiating Eq. (3) and linearizing with respect to $\delta \theta_{n,n+1}$ gives in the region $ns\leq z\leq (n+1)s$

$$\frac{\partial J_c(x,t)}{\partial t} = J_{n,n+1}(r) \cos \theta_{n,n+1}^0(r) \frac{\partial \delta \theta_{n,n+1}(r,t)}{\partial t}.$$  

(5)

In addition, we now assume that $E_z$ is slowly varying along the $z$ direction, so that $(E_z(r,t))_{n,n+1}$ can be replaced by $E_z(r,z,t)$. Finally, we combine Eqs. (2), (4), and (5), and seek a solution of the form $E_z(r,z,t) = \text{Re} \left[ E(r,z,\omega) \exp(i\omega t) \right]$. In the region $ns\leq z\leq (n+1)s$, $E(r,z,\omega)$ is then found to satisfy the differential equation

$$\left( \nabla^2 + \frac{2 \partial^2}{\partial z^2} + \frac{\epsilon_0 \omega^2}{c^2} \right) E_z(r,z,\omega) = \frac{e^* s J_{n,n+1}(r)}{\hbar c^2} \cos \theta_{n,n+1}^0(r) E_z(r,z,\omega).$$

(6)

If we neglect the spatial variation of $E_z$ in the $z$ direction, this differential equation may be written [for $ns\leq z\leq (n+1)s$]

$$\left( \nabla^2 + \epsilon(r,n,\omega) \frac{\omega^2}{c^2} \right) E_z(r,\omega) = 0,$$

(7)

where $\epsilon(r,n,\omega) = \epsilon_0 [1 - \omega^2 n(r,n)/\omega^2]$, and

$$\omega^2 n(r,n) = [8 \pi^2 c s / (\epsilon_0 \Phi_0)] J_{n,n+1}(r) \cos \theta_{n,n+1}^0(r).$$

(8)

Equation (7) is equivalent to the form previously discussed by Bulaevskii and co-workers. This equation is also formally equivalent to the Schrödinger equation for a particle in a two-dimensional random potential proportional to $\omega^2 n(r,n)$. In general, $\omega^2 n(r,n)$ depends on both transverse position $r$ and $n$, and is also a thermodynamically fluctuating quantity, since both $J_{n,n+1}(r,T)$ and $\cos \theta_{n,n+1}^0(r,T)$ have these dependencies. We will estimate it by replacing $\omega^2 n(r,n)$ by $\langle \omega^2 n(r,n) \rangle = \omega_{pl}^2$, where $\langle \cdots \rangle$ denotes both a statistical average in the canonical ensemble and an average over $r$ and $n$. This may be reasonable at high magnetic fields, where the vortex cores overlap to such an extent that the effects of the randomness in the potential are small.

At sufficiently high fields, it is adequate to expand $\psi_n(r)$ in LLL states of the $n$th layer. With the gauge choice $A = -Byz$, this expansion may be written

$$\psi_n(r) = \left( \frac{\sqrt{3} a_c^2(T)}{4b^2} \right)^{1/4} \sum_k c_{kn} \exp \left( ikx - (y-kl)^2 / 2l^2 \right),$$

(9)

Here $l = \sqrt{\Phi_0/(2\pi B)}$, $a_c(T) = a_c(T)[1-B/H_{c2}(T)]$, $H_{c2}(T)$ is defined by $\alpha(T) + \alpha H_{c2}(T)/(2m_{ab}c) = 0$, and

$$b = 2\pi \kappa^2 [\epsilon^* / (m_{ab}c) + \kappa^2] \exp(\mu_0/2) [\lambda_{ab}(0)/\epsilon^*_0(0)]$$

[where $\lambda_{ab}(0)$ is the zero-temperature penetration depth and coherence length], and finally, $T_0$ is the mean-field transition temperature for $B=0$. The index $k = 2\pi pl/L_x$ labels the different LLL’s in a given layer, with $0 \leq p \leq N_{pl} - 1$, where $N_{pl} = N_x N_y = L_x L_y / (2\pi l^2)$ is the number of vortices in each layer, and $L_x = (4\pi / \sqrt{3})^{1/2} N_x$, $L_y = (4\pi / \sqrt{3})^{1/2} N_y$ are the linear dimensions of the system in the $x$ and $y$ directions. The coefficients $c_{kn}$ are the fluctuating amplitudes of the various LLL’s in different layers.

With this choice of basis, the final expression for $\omega_{pl}^2$ is

$$\omega_{pl}^2 = \omega_{pl}^2 \left( \frac{\beta_A}{2 N_\phi N_z} \sum_{k,n} (c_{kn} c_{kn+1}^* + c.c.) \right),$$

(10)

where $\langle \cdots \rangle_{\psi}$ denotes a thermal average in the canonical ensemble, $\omega_{pl}^2 = \sqrt{(H_{c2}(T) - B_c c^2/\epsilon^*_0)^2 / \epsilon^*_0 \kappa^2 \gamma^4 \hbar^2}$ is the mean-field Josephson plasmon frequency, and $\beta_A = 1.159 595 \ldots$ is the Abrikosov ratio.

III. RESULTS

A. Numerical results from Monte Carlo simulations

We evaluate $\omega_{pl}^2$ in an $N_x \times N_y \times N_z$ system with periodic boundary conditions, using $N_z = 8$ layers, and choosing $N_x = N_y$ to accommodate exactly $N_\phi = 64$ vortex pancakes in a triangular lattice. We use the following parameters for Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_{8+\delta}$: $T_{c0} = 8.5 \text{ K}$, $d H_{c2}(T)/dT = -5.2 \times 10^3 \text{ Oe/K}$, $s = 15 \text { Å}$, $d = 4 \text { Å}$, $\gamma = 150$, and $\kappa = 100$. We carry out the required average for various $B$ and $T$ both above and below the melting temperature $T_{MC}(B)$. To carry out the simulation, for each $T$ and $B$, we average over $3 \times 10^5$ Monte Carlo (MC) sweeps through the lattice, after discarding the first $2 \times 10^4$ sweeps for equilibration. At each field, we gradually warm up from an initial triangular lattice, using the final configuration for a lower $T$ as the initial one for the next higher $T$. This procedure may correspond to the “field-cooled” measurements of Ref. 3. By contrast, a “zero-field-cooled” measurement (i.e., one in which the system is first cooled to the desired temperature in zero field, after which the field is turned on at fixed $T$), might well give different, possibly nonequilibrium, results.

Figure 1(a) shows the calculated quantity $\sqrt{\epsilon_0 \omega_{pl}(B,T)}$ (which is independent of $\epsilon_0$) for Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_{8+\delta}$. At all
the melting curve nor the lines of constant LLL approximation is not reliable at low B, as shown in Refs. 1-3, but in contrast to those, never in experiments. Qualitatively, our curves somewhat resemble those found at 105-220 GHz near and above the melting temperature, relative to the clean material at the same temperature. There is no obviously special behavior at the liquid state, which is valid beyond the LLL approximation. Our result is based on the fact that $\gamma_{zz}$ vanishes in the liquid. We start with Eq. (8), the average of which gives the $\omega_{pl}$ in either the vortex solid or the vortex liquid phase. This expression, being independent of the LLL approximation, is applicable, in
principle, at both low fields as well as high fields, and in
disordered as well as ordered systems.

To obtain our result, we use a general formula for \(\gamma_{zz}^{22}\) to
cast the average of Eq. (8) into a different form. Using the
vanishing of \(\gamma_{zz}\) in the liquid state, together with Eq. (8) of
Ref. 22, we find that in the flux liquid state, \(\omega_{p}^{2}(B,T)\) can be
expressed in the form

\[
\omega_{p}^{2}(B,T) = \frac{4\pi \langle j_{z}^{2} \rangle_{T}}{\epsilon_{0} V k_{B} T}. \tag{12}
\]

Here \(J_{z} = \int d^{3}x j_{z}(x)\) is the volume integral of the supercur-
current density \(j_{z}(x)\) in the \(c\) direction, and \(V\) is the system
volume. To obtain this expression, we have used the fact that
\(J_{z} = \sum_{n} \int d^{3}r j_{z}(\mathbf{r})\sin \theta_{n+1}(\mathbf{r}) d\mathbf{r}\). Equation (8) of Ref. 22
can then be applied directly once this integral is discretized
on a suitable lattice, provided that thermal amplitude fluctua-
tions in \(J_{n+1}(\mathbf{r},t)\) can be neglected.

Equation (12) may be further simplified if we make the
approximation that \(\langle j_{z}(\mathbf{r}) j_{z}(\mathbf{r}') \rangle\), in the liquid, depends only
on \(\mathbf{r} - \mathbf{r}'\). (In fact, this assumption is exact in the clean sys-
tem, but not in the dirty system, where impurities break the
translational invariance.) This assumption leads to the form

\[
\omega_{p}^{2}(B,T) \sim \frac{4\pi \epsilon_{0} v_{c}}{\epsilon_{0} B k_{B} T} \frac{\langle j_{z}^{2}(0) \rangle}{\langle j_{z}(0) \rangle}.
\]

Equation (14) is similar to Koshelev’s expression,\(^6\) obtained
from a high-temperature expansion in the liquid state of a
clean material, but differs slightly because we have estimated
the relevant correlation function using different approxima-
tions.

According to Eq. (14), \(\omega_{p}\) should vary as \(1/\sqrt{B}\) at fixed
temperature everywhere in the flux liquid state. This is pre-
cisely the field dependence that is observed experimentally
in both the liquid and the solid phases.\(^5\) We do not have a
similar argument for the behavior of \(\omega_{p}^{2}\) in the solid phase,
however.

IV. DISCUSSION AND CONCLUSIONS

Using a Monte Carlo method, we have calculated
\(\omega_{p}^{2}(B,T)\) for a model of Bi\(_{2}\)Sr\(_{2}\)Ca\(_{2}\)Cu\(_{2}\)O\(_{8+\delta}\) in the LLL
approximation both with and without the columnar defects, in
both the flux solid and flux liquid state. In both cases, we

FIG. 2. (a) Calculated helicity modulus \(\gamma_{zz}(B,T)\) (in units of
10\(^{-16}\) cm) and (b) the product \(\sqrt{\epsilon_{0} \omega_{p}^{2}(B,T)}\) (in units of 100 GHz)
as functions of \(T\) at \(B=2\) T, in clean Bi\(_{2}\)Sr\(_{2}\)Ca\(_{2}\)Cu\(_{2}\)O\(_{8+\delta}\) and with various densities \(D\) of randomly distributed columnar defects par-
allel to the \(c\) axis. For each defect density, the calculated points are
averaged over ten disorder realizations; the error bars are rms de-
viations over the disorder. For the clean system, the lines are quasi-
Hermite spline fits; for the dirty systems, straight-line interpola-
tions.
find, as expected, that $\omega_{pl}^2$ is nonzero in the liquid phase. Randomly distributed columnar defects are found to raise the melting temperature above that of the clean system $T_m(B)$. They also appear to convert melting from a first-order phase transition to a continuous transition, as expected from previous work.\(^\text{18}\) Likewise, in the presence of columnar disorder, $\omega_{pl}^2(B,T)$ appears to vary continuously with temperature at fixed $B$, while without it, $\omega_{pl}^2$ is discontinuous across the melting line.

Besides our numerical results, we have also obtained a general analytical expression for $\omega_{pl}^2$ in the flux liquid state, in terms of thermal fluctuations of current density. The expression is valid in both clean and dirty systems, and is not limited to the LLL approximation. It strongly suggests that $\omega_{pl}^2$ is nonzero everywhere in the liquid state. If we further approximate this exact result, we obtain an expression very similar to Koshelev’s result from a high-temperature expansion.\(^\text{6}\) This expression predicts, in agreement with experiment,\(^\text{4}\) that $\omega_{pl}(B,T) \sim 1/\sqrt{B}$ in the flux liquid state.

Our numerical results agree qualitatively with experiment in some respects, but not all. In the absence of disorder, our calculated $\omega_{pl}^2(B,T)$ decreases monotonically with $T$, in agreement with field-cooled experiments.\(^\text{1}\) But in contrast to experiment, the calculated $\omega_{pl}^2$ for clean systems is discontinuous across the melting curve, and the contours of constant $\omega_{pl}^2$ do not cross the melting curve. When disorder is introduced (in our case, random columnar disorder), $\omega_{pl}^2(B,T)$ varies continuously with $T$ across the glass transition. Presumably, the experimental curves are continuous because the experimentally studied materials undergo a continuous glass transition rather than the first order melting transition expected for the clean system. Even in our systems with quenched columnar disorder, the calculated contours of constant $\omega_{pl}^2$ appear not to cross the glass transition line.

Quenched point disorder should produce similar continuous behavior of $\omega_{pl}^2$ across the melting line, since such disorder is believed to convert the first-order melting curve to a continuous vortex-glass transition.\(^\text{19}\) We have not studied this case numerically purely for convenience: since random columnar disorder produces a much more dramatic effect on the melting curve, it hence is easier to study numerically.

A recent paper by Glazman and Koshelev\(^\text{21}\) suggests that, in a highly anisotropic superconductor such as Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_{8+\delta}$, there are actually two transitions in the clean limit: a lower melting temperature where the phase stiffness $\gamma_z$ vanishes, and an upper one where the layers become decoupled. Although the lower temperature corresponds to a sharp phase transition, the upper one is probably a crossover rather than a sharp phase transition. In the presence of point disorder, these transitions exhibit more complex behavior,\(^\text{24}\) while the nature of the upper transition in the presence of line disorder is unclear.

Our results suggest that, in both clean samples and ones with line disorder, there is indeed a melting transition, followed at higher temperatures by a decoupling transition. The phase stiffness in the $c$ direction vanishes at the lower transition, while $\omega_{pl}^2$ becomes very small at the upper transition, which might conceivably be identified with the Glazman-Koshelev decoupling transition in either the ordered or line-disordered system. We believe that the upper transition, at least in our LLL approximation, is a crossover rather than a sharp phase transition in both the ordered and line-disordered samples.

The Glazman-Koshelev theory leads to a so-called decoupling field, which is in the range of 1 kOe for Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_{8+\delta}$. Near that field, the melting curve could exhibit some anomalies, even in the clean system. However, such behavior would be very difficult to detect in our present LLL approximation, which is appropriate for high fields, probably above 1 T. For the same reason, we are unable to use the LLL approximation to study the behavior of the melting curve in the clean system at low magnetic fields. (The melting curve of clean Bi$_2$Sr$_2$Ca$_2$Cu$_2$O$_{8+\delta}$, as calculated in the LLL, has been further discussed in Ref. 25.)

Finally, at low magnetic fields, $\omega_{pl}(B,T)$ has been reported to vary as $1/\sqrt{B}$. Our exact expression for $\omega_{pl}^2$ in the flux liquid state does lead to such a variation when certain further approximations are made [cf. Eq. (14)]. This expression is applicable even to the moderately low-field regime where such behavior is reported.\(^\text{4}\)

The present work leaves open a number of future problems. For example there should be a spread in $\omega_{pl}^2$, rather than a single sharp line, arising from fluctuations in local magnetic field; this spread should be included in the calculations. There will be further damping from the normal interlayer current in parallel with the supercurrent. Also, it is possible that the electrodynamics underlying our calculation for $\omega_{pl}^2$ are oversimplified (neglecting, for example, the $z$ dependence of $E_z$). Finally, of course, it will be important to carry out calculations beyond the LLL approximation, and to connect $\omega_{pl}^2$ more completely to the details of the phase diagram in the $H$-$T$ plane. We hope to return to some of these points in future work.

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14 $T_M$ has previously been determined for a clean lattice using the criterion that the helicity modulus component $\gamma_{zz}$, i.e., the $z$ component of the superfluid density, should vanish at $T_M(B)$ (Ref. 7). As shown in that paper, the sharp Bragg spots of the vortex lattice also vanish at this same temperature in a clean lattice. The transverse components of the helicity modulus tensor are zero even in the solid phase of the clean lattice, because, in the absence of pinning, the lattice can slide freely in the $ab$ plane.


21 We briefly discuss the numerical uncertainties in the values of $\omega_{pl}(B,T)$ calculated in the liquid state. First, in the clean system, the statistical uncertainties arising from the Monte Carlo procedure, at any given temperature, are probably less than 10% even in the liquid state—that is, an extremely long Monte Carlo run on a system of the same size would give a number within 10% of the value shown in the graph. The statistical uncertainties in the disordered system are not much larger than this for any given realization; the seemingly large uncertainties are mostly due to the variation in $\omega_{pl}$ from one realization to another in our rather small system. Thus, our numerical results indicate with a high degree of certainty that $\omega_{pl}^2$ is nonzero in much of the liquid phase, for both clean samples and samples with random columnar disorder.


