Fluctuation effects on critical behavior of Josephson-junction arrays

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This paper investigates the effects of fluctuations on critical behavior of two-dimensional periodic arrays of coupled Josephson junctions. Both random fluctuations of the external magnetic field and quantum fluctuations are considered. The standard replica method is used to deal with fluctuations of the magnetic field, and it is shown that weak fluctuations are irrelevant whereas strong fluctuations destroy superconductivity. In the case of medium fluctuations, strong dependence on the lattice structure is found: They are irrelevant on a square lattice while they destroy the effect of the magnetic field on a triangular lattice. Quantum fluctuations are treated with use of a variational method. It is shown that they affect the renormalization-group equation in a significant way and lead to a low-temperature reentrant transition into a normal phase. Also shown is that sufficiently strong quantum fluctuations destroy superconductivity at all temperatures. Limitation of this analysis and relevance to experiments are discussed.

I. INTRODUCTION

An external magnetic field has interesting effects on two-dimensional (2D) superconductors, which exhibit the Kosterlitz-Thouless (KT) transition. In the case of homogeneous superconducting films it produces mutually repulsive vortices, which form a regular flux lattice at zero temperature. In the case of periodic arrays, however, new effects are expected to be observed since the interaction between the natural periodicity of the flux lattice and the underlying array periodicity produces a kind of commensurate-incommensurate effect as the external field is varied.

Experiments on 2D arrays of coupled Josephson junctions indeed show novel behavior such as the periodic variation of resistivity with the magnetic field. In the high-capacitance limit these arrays can be described by uniformly frustrated 2D XY models, which possess discrete symmetries in addition to the underlying continuous U(1) symmetry. Revealed by a number of works on these models is a wide variety of critical behavior according to values of the frustration of $f$. For example, the transition temperature of the commensurate system ($f = m/n$, with $m$ and $n$ relatively prime) has been indicated to depend on $n$, while there is evidence of no phase transition in the incommensurate system (with $f$ an irrational number). This implies that the critical behavior as well as the symmetries of the system are highly discontinuous functions of $f$, since an arbitrarily small change in $f$ would turn a commensurate system into an incommensurate one and vice versa. Here a curious question arises about the effects of random fluctuations of $f$ around a rational value on the critical behavior of that system. This question is quite relevant to real experiments, since the frustration $f$ relates to the external magnetic field in the case of Josephson-junction arrays, whose fluctuations are inevitable. It is therefore of interest, for example, to investigate whether random fluctuations of the field destroy the transition of a peculiar type in the fully frustrated system ($f = 1/2$) predicted by recent renormalization-group (RG) studies.

Another interesting topic in connection with experiments, where the junction capacitance is in fact finite, is the effects of quantum tunneling between junctions. It was pointed out that these charging effects would suppress superconductivity in granular materials. In particular, the possibility of reentrance into the normal state at low temperatures due to these quantum fluctuations has attracted interest even in the absence of the external field, i.e., in the absence of the frustration ($f = 0$). Results from mean-field calculations and self-consistent harmonic approximations are not quite conclusive, while at zero temperature the system can be transformed into a three-dimensional classical XY model with two space dimensions and one time dimension, and therefore is expected to display behavior similar to that of $^{4}$He. Recent RG analysis seems to favor the existence of such a reentrant transition, but this is doubtful because of the perturbative nature in dealing with quantum fluctuations. Monte Carlo simulations have also been carried out on this quantum problem, leading to results which support its existence. Since there always exist quantum fluctuations in real experiments, it is of interest to study their effects on the critical behavior of Josephson arrays in the presence of a magnetic field.

The purpose of this paper is to investigate the effects of these fluctuations on the critical behavior of Josephson-junction arrays. Random fluctuations of the magnetic field, which have a Gaussian distribution, are interpreted in terms of those of the frustration or the bond angle in the corresponding frustrated XY model. We then use the standard replica method to obtain the effective Hamiltonian for this random system, from which the Landau-Ginzberg-Wilson (LGW) Hamiltonian is derived through the use of a Hubbard-Stratanovich transform. Thus, it is suggested that as in the case of the "purely" frustrated XY model, for some simple (mean) values of $f$, this "randomly" frustrated XY model may be decomposed into...
coupled \( XY \) models but with lattice anisotropy. It is shown that sufficiently weak fluctuations of the field do not affect the critical behavior, whereas strong fluctuations make the system effectively one dimensional and therefore would destroy the superconducting phase. On the other hand, the effects of medium fluctuations, which will be made precise in Sec. II, depend strongly on the lattice structure. On a square lattice they are irrelevant; the fully frustrated system would exhibit a single transition into the superconducting phase, which is a combination of a KT-like one and an Ising-like one even in the presence of the field fluctuations. On a triangular lattice, however, they in general decouple the critical modes and lead to a conventional (unfrustrated) \( XY \) model. Therefore, for example, a conventional KT transition is predicted for the fully frustrated system. To investigate the effects of quantum fluctuations we next consider the Hamiltonian which includes the diagonal charging energy as well as the Josephson energy in the presence of the magnetic field.

The partition function of this quantum system is formulated in terms of the Feynman path integral, which is evaluated through the use of the variational principle. Although this variational method is also semiclassical in nature, it in general gives better results than the perturbation expansion. It is shown that within this approximation the Josephson coupling between neighbors is renormalized in a significant way by quantum fluctuations. These effects on the RG equations are analyzed to show the existence of a reentrant transition in fully frustrated systems as well as in unfrustrated systems.

There are four sections in this paper. Section II deals with fluctuations of the external magnetic field, while Sec. III is devoted to the investigation of quantum fluctuations. The main results are summarized and a discussion is given in Sec. IV.

II. FLUCTUATIONS OF THE MAGNETIC FIELD

The uniformly frustrated \( XY \) model is described by the Hamiltonian \( \beta = 1/kT \)

\[
-\beta H = BJ \sum_{(\alpha \beta)} \cos[\phi(\alpha) - \phi(\beta) - A_{\alpha \beta}] ,
\]

(2.1)

where \( A_{\alpha \beta} \) is the bond angle such that the plaquette sum is constant over the whole lattice, \( \sum A_{\alpha \beta} = 2\pi f \). With the identification

\[
A_{\alpha \beta} = \frac{2e}{\hbar c} \int_{r} A \cdot d1 ,
\]

(2.2)

the Hamiltonian given by Eq. (2.1) describes arrays of Josephson junctions in the high-capacitance limit. In Eq. (2.2), \( A \) is the vector potential, which may be taken to be that of a uniform transverse magnetic field \( B \) in the limit of large penetration depth. The uniform frustration \( f \) is then related to \( B \) via

\[
f = BA_{p} / \Phi_{0} ,
\]

(2.3)

where \( A_{p} \) is the area of a plaquette and \( \Phi_{0} = \hbar c / 2e \) is the flux quantum.

We now consider random fluctuations of the magnetic field, or equivalently, random fluctuations of \( f \) around a mean value \( f' \), which have a Gaussian distribution

\[
P(f) = (2\pi\Delta)^{-1/2} \exp[-(f - f')^{2} / 2\Delta^{2}] .
\]

(2.4)

This implies that \( A_{\alpha \beta} \) defined by Eq. (2.2) also fluctuates randomly. To deal with this randomness we use the standard replica method and obtain the effective Hamiltonian

\[
-\beta H_{\text{eff}} = \ln\left( \exp\left[ -\sum_{\alpha=1}^{n} \beta H_{\alpha} \right] \right)
\]

\[
= \beta J \sum_{\alpha} \sum_{(\alpha \beta)} \left( \cos[\phi(\alpha) - \phi(\beta) - A_{\alpha \beta}] \right) + \cdots ,
\]

(2.5)

where \( \alpha \) is a replica index and the average is to be taken with respect to the distribution given by Eq. (2.4). In general, higher cumulants (as indicated by the ellipsis) in Eq. (2.5) are irrelevant and will be neglected hereafter.

A. Square lattices

On a square lattice the bond angle given by Eq. (2.2) can be expressed in terms of \( f' \) and it is straightforward to evaluate the average in Eq. (2.5):

\[
\langle \cos[\phi(\alpha) - \phi(\beta) - A_{\alpha \beta}] \rangle
\]

\[
= \left[ \cos[\phi(\alpha) - \phi(\beta) - \bar{A}_{\alpha \beta}] \right] \text{ for } r' = r \pm \hat{x}
\]

\[
= e^{-2\pi^{2}a^{2}k^{2}} \cos[\phi(\alpha) - \phi(\beta) - \bar{A}_{\alpha \beta}] \text{ for } r' = r \pm \hat{y},
\]

(2.6)

where \( \bar{A}_{\alpha \beta} \) is the (nonfluctuating) mean value of \( A_{\alpha \beta} \) and Eq. (2.4) has been used. Thus the effective Hamiltonian in Eq. (2.5) takes the form

\[
-\beta H_{\text{eff}} = \beta \sum_{\alpha} \sum_{(\alpha \beta)} J_{\alpha \beta} \cos[\phi(\alpha) - \phi(\beta) - \bar{A}_{\alpha \beta}] ,
\]

(2.7)

with

\[
J_{\alpha \beta} = \begin{cases} J \text{ for } r' = r \pm \hat{x} \\ e^{-2\pi^{2}a^{2}k^{2}} J \text{ for } r' = r \pm \hat{y}, \end{cases}
\]

where the bar over \( A_{\alpha \beta} \) has been omitted for simplicity. To obtain the critical modes, we now consider the Fourier transform of the interaction matrix

\[
P_{qq'} = N^{-1} \sum_{r,r'} e^{-i(qr - q' r')} J_{rr'} e^{-iA_{\alpha \beta}},
\]

(2.8a)

where \( N \) is the number of the lattice sites and \( R_{q'q} \) is given by

\[
R_{q'q} = \int \frac{d^{d}k}{(2\pi)^{d}} e^{i(q' - q)k} P(qk) ,
\]

(2.8b)
In the thermodynamics limit \((N \to \infty)\), Eq. (2.8b) reduces to a simple form
\[
R_{q_1 q_1'} = 2 \cos q_1 \delta_{q_1 q_1'} + N^{-1} \sum_x e^{-i(q_1 - q_1') + 2\pi f x} e^{-2\pi^2 \Delta x^2} + N^{-1} \sum_x e^{-i(q_1 - q_1') - 2\pi f x} e^{-2\pi^2 \Delta x^2}.
\]

(2.8b)

with
\[
p = (8\pi \Delta)^{-1/2} \Phi(\sqrt{2} \pi \Delta) \quad (0 \leq p \leq 1),
\]

where terms of \(O(N^{-1})\) have been neglected. Here, \(\Phi(x)\) is the probability integral (also known as the error function) defined to be
\[
\Phi(x) = \frac{1}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt,
\]

and \(\Delta\) has been assumed to scale in the thermodynamic limit as \(\Delta = \Delta/N\) with finite \(\Delta\), which defines medium fluctuations. If fluctuations are weak enough that \(\Delta \to 0\) in the thermodynamic limit, then we have \(p \to 1\) in Eq. (2.9). This implies that the system behaves as if there were no fluctuations at all. Thus, essentially a pure system is obtained. On the other hand, fluctuations strong enough that \(\Delta \to \infty\) in the thermodynamic limit leads to \(p \to 0\) in Eq. (2.9). This would make the system effectively one dimensional and therefore destroy the superconducting phase.

To investigate further the interesting case of medium fluctuations \((0 < p < 1)\), we consider the fully frustrated system, which gives Eq. (2.9) in the form of a \(2 \times 2\) matrix
\[
R = 2 \begin{pmatrix} \cos q_1 & p \cos q_2 \\ p \cos q_2 & -\cos q_1 \end{pmatrix}.
\]

(2.10)

The dominant eigenvalue is
\[
\lambda_d = 2(\cos^2 q_1 + p^2 \cos^2 q_2)^{1/2},
\]

(2.11)

which reduces its largest value \(\lambda_d = 2(1 + p^2)^{1/2}\) at \(Q_1 = (0, 0)\) and \(Q_2 = (0, \pi)\). The critical modes are thus the same as those in the pure system \((p = 1)\). Consequently, we get the same LGW Hamiltonian as that for the pure system obtained in Ref. 8, except for the factor \(p^2\) in the \(q_2^2\) term. The phase-only approximation, which is expected to be accurate at temperatures well below the mean-field transition temperature,\(^2\) then allows a generalization of the LGW Hamiltonian of the form
\[
-\beta H_{\text{eff}} = -\sum_{a=1}^N \beta H_a
\]

(2.12)

\[
-\beta H_a = \sum_{(r,r')} K_{rr'} \left[ \cos[\theta^o(r) - \theta^o(r')] + \cos[\theta^o(r) - \theta^o(r')] \right]
\]

\[
+ h \sum_r \cos 2[\theta^o(r) - \theta^o(r')],
\]

with lattice anisotropy
\[
K_{rr'} = \begin{cases} \beta J/\sqrt{2} & \text{for } r' = r \pm \hat{x} \\ \beta p^2 J/\sqrt{2} & \text{for } r' = r \pm \hat{y}. \end{cases}
\]

Thus, we have decomposed the Hamiltonian into coupled \(XY\) Hamiltonians with lattice anisotropy.

If we perform the same renormalization procedure for the system described by the effective Hamiltonian in Eq. (2.12) as that for the pure system, we would obtain the same scaling equations as those for the pure system, two coupled \(XY\) models with lattice anisotropy since replicas would not mix during the course of renormalization.\(^11\) It is also obvious that such lattice anisotropy is irrelevant to the critical behavior unless \(p = 0\),\(^20\) while for \(p = 0\) (strong fluctuation), we obtain the two coupled \(XY\) chains, leading to no phase transition. This feature holds for other values of \(f\), and it is concluded that on a square lattice, random fluctuations of the magnetic field are in general irrelevant unless they are sufficiently strong.

B. Triangular lattices

The average in Eq. (2.5) is evaluated on a triangular lattice:

\[
\langle \cos[\phi^o(r) - \phi^o(r') - A_{rr'}] \rangle = \begin{cases} \cos[\phi^o(r) - \phi^o(r') - A_{rr'}] & \text{for } r' = r \pm \hat{x}, \\ e^{-\pi \Delta (4x + 1)/2} \cos[\phi^o(r) - \phi^o(r') - A_{rr'}] & \text{for } r' = r + \frac{1}{2} \hat{x} \pm \frac{\sqrt{3}}{2} \hat{y}, \\ e^{-\pi \Delta (4x - 1)/2} \cos[\phi^o(r) - \phi^o(r') - A_{rr'}] & \text{for } r' = r - \frac{1}{2} \hat{x} \pm \frac{\sqrt{3}}{2} \hat{y}. \end{cases}
\]

(2.13)

The effective Hamiltonian takes the same form as that on a square lattice given by Eq. (2.7), except that \(J_{rr'}\) is now determined by Eq. (2.13). The Fourier transform of the interaction matrix is again given by Eq. (2.8a) with the expression for \(R_{q_1 q_1'}\)

\[
R_{q_1 q_1'} = 2 \cos q_1 \delta_{q_1 q_1'} + 2pe^{i\sqrt{3}q_1/2} \cos(q_1/2 + \pi f)\delta_{q_1 + 4\pi f, q_1'}
\]

\[
+ 2pe^{-i\sqrt{3}q_1/2} \cos(q_1/2 - \pi f)\delta_{q_1 - 4\pi f, q_1'},
\]

(2.14)
where \( p \) is in this case given by
\[
p = \left(32\pi^2ight)^{-1/2} \Phi(2\sqrt{2\pi^2}) \quad (0 \leq p \leq 1)
\]
We consider the fully frustrated system for which Eq. (2.14) takes the form
\[
R = 2 \begin{pmatrix}
\cos q_1 & -2ip \sin(q_1/2) \sin(\sqrt{3}/2) q_2 \\
2ip \sin(q_1/2) \sin(\sqrt{3}/2) q_2 & \cos q_1
\end{pmatrix}
\]
(2.15)
The dominant eigenvalue is
\[
\lambda_q = 2 \cos q_1 + 4p | \sin(q_1/2) \sin(\sqrt{3}/2) q_2 |
\]
and reaches its largest value \( \lambda_Q = 2 + p^2 \) at
\[
Q_{1,2} = (\pm 2 \sin^{-1}(p/2), \pi/\sqrt{3})
\]
These two critical modes depend on \( p \), and differ from those in the pure system \( p = 1 \), indicating different critical behavior. In particular, except for some rather special values of \( p, 2 \sin^{-1}(p/2) \) is not commensurate with \( \pi \), i.e., there do not exist integers \( m \) and \( n \) such that
\[
2 \sin^{-1}(p/2) = m \pi / n.
\]
This suggests that there is no coupling of the form \( \cos [\theta_1(r) - \theta_2(r)] \) between the two critical modes, leading to just two (uncoupled) XY models. Thus, implied is a conventional KT transition, since replicas again do not mix and lattice anisotropy is irrelevant. In this case, random fluctuations of the field destroy the Ising-like transition associated with the discrete symmetry of the pure system. This behavior, which is vastly different from that on a square lattice, results from the fact that on a triangular lattice the off-diagonal elements of the interaction matrix \( R \) depend on both \( q_1 \) and \( q_2 \), and is expected to prevail for other values of \( f \). For \( p = 0 \), the system again becomes effectively one dimensional, leading to no phase transition. Hence it is concluded that on a triangular lattice, random fluctuations of the field are relevant unless they are sufficiently weak.

III. QUANTUM FLUCTUATIONS

We now consider the Hamiltonian
\[
-HH = -\frac{\beta u}{2} \sum_r n_r^2 - \beta J \sum_{(rr')} \cos[\phi(r) - \phi(r') - A_{rr'}]
\]
(3.1)
where the first term corresponds to the charging energy. In that term the operator \( n_r \) measures the number of excess Cooper pairs on the junction located at \( r \) and is conjugate to the phase \( \phi(r) \), i.e., \( n_r = i \partial \phi(r) / \partial \phi(r) \). \( u \) is in general related to the inverse of the capacitance of the arrays. In Eq. (3.1) we have used the diagonal approximation for the charging energy, which is expected to be reasonable for close-packed systems.\(^{14}\) \( u \) is then given by \( u = 4e^2/C \).\(^{16}\) In the high-capacitance limit, we have \( u = 0 \) and recover the "classical" frustrated XY models.

In the more realistic case of the finite capacitance, however, \( u \) is nonzero and the quantum effects become relevant.

A convenient way to deal with the quantum Hamiltonian of the form given by Eq. (3.1) is the Feynman path integral formulation.\(^{19}\) The partition function of the quantum system described by the Hamiltonian in Eq. (3.1) can be expressed in terms of a Feynman path integral, which may be evaluated via the variational principle. The detailed procedure can be found in literature\(^{19}\) and will not be repeated here. The result may be summarized as follows: we can regard this quantum system as approximately classical, i.e., \( u = 0 \), if we use an effective potential \( U \),
\[
U(\{\phi(r)\}) = \left[ \frac{6}{\pi \beta u} \right]^{N/2} \int_{-\infty}^{\infty} d\zeta(r) e^{-6\zeta(r)/\beta u}
\times V(\{\phi(r) + \zeta(r)\})
\]
(3.2)
in place of
\[
V(\{\phi(r)\}) = -J \sum_{(rr')} \cos[\phi(r) - \phi(r') - A_{rr'}]
\]
The integral in Eq. (3.2) can be performed straightforwardly to give
\[
U(\{\phi(r)\}) = -Je^{-\beta u/12} \sum_{(rr')} \cos[\phi(r) - \phi(r') - A_{rr'}]
\]
(3.3)
which shows that \( J \), the Josephson coupling between neighbors, is renormalized to \( Je^{-\beta u/12} \) by quantum fluctuations.\(^{21}\) Thus, except for this peculiar renormalization, we obtain essentially the same Hamiltonian as that for the classical system. For example, in the phase-only approximation the Hamiltonian given by Eq. (3.3) with \( f = \frac{1}{2} \) (fully frustrated system) may be decomposed into two coupled XY Hamiltonians\(^{8}\)
\[
-HH = -Ke^{-K_\alpha} \sum_{(rr')} \left[ \cos[\theta_1(r) - \theta_1(r')] + \cos[\theta_2(r) - \theta_2(r')] \right]
+h \sum_r \cos[\theta_1(r) - \theta_2(r)]
\]
(3.4)
where the effective interaction \( K \) is given by
\[
K = \begin{cases}
\beta J / \sqrt{2} & \text{for square lattices,} \\
\beta J / 2 & \text{for triangular lattices,}
\end{cases}
\]
and the dimensionless parameter \( \alpha = \beta u / 12K \sim u / J \) measures the ratio of the charging energy to the Josephson energy. The mode-coupling field \( h \) depends on the quantity \( Je^{-\beta u/12} \), and \( p = 2(3) \) for square (triangular) lattices.

We now consider the renormalization of the Hamiltonian in Eq. (3.4) by both vortices and the mode-coupling field. This has been already done for the classical case
and we just need to substitute $K e^{-K \alpha}$ for $K$ in the RG equations for the classical system. Thus, we obtain the scaling equations for the quantum system ($\alpha \neq 0$), which reads

$$\frac{dK}{dl} = \frac{e^{K \alpha}}{1 - K \alpha} \left[ \frac{1}{2} \tilde{y}^2 - \left( K^2 e^{-2K \alpha} + (K e^{-K \alpha} - \tilde{K} e^{-K \alpha})^2 \right) \right],$$

$$\frac{d\tilde{K}}{dl} = \frac{e^{K \alpha}}{1 - \tilde{K} \alpha} (\tilde{y}^2 - \tilde{K}^2 e^{-2K \alpha} \tilde{y}^2),$$

$$\frac{dy}{dl} = (2 - \pi K e^{-K \alpha}) y,$$

$$\frac{d\tilde{y}}{dl} = \left[ 2 - \frac{p^2 e^{K \alpha}}{2 \pi \tilde{K}} \right] \tilde{y}.$$

(3.5)

As is for the classical case, $\tilde{K}$ is equal to $K$ initially; however, $\tilde{K}$ becomes different from $K$ as renormalization proceeds, generating the off-diagonal coupling between the two modes. In Eq. (3.5), $y$ is the vortex fugacity while $\tilde{y}$ is the coupling charge fugacity, given by $\tilde{y} = p^{1/2} h / 2$ for small $h$.

The scaling equations given by Eqs. (3.5) show that for $\alpha < \alpha_c \equiv \pi/2e$, vortices are irrelevant only in the region $K_1 < K < K_2$, where $K_1$ and $K_2$ are the two solutions of the equation $2 - \pi K e^{-K \alpha} = 0$. In other words, the spin-wave fixed line is unstable to vortices at low temperatures ($K > K_2$) as well as at high temperatures ($K < K_1$). Physically, this implies the existence of a low-temperature paramagnetic phase in addition to the high-temperature paramagnetic phase and the ordered phase in between.

As the temperature is lowered below the upper transition temperature $T_{c2}$, the junction array will undergo a phase transition into an ordered state, where the phases lock together, leading to zero resistivity. This transition would be a combination of a KT-like one and an Ising-like one; in the ordered phase there exists long-range order for chirality associated with the discrete symmetry of the system, as well as algebraic order for the XY spins, i.e., phases.

If the temperature is cooled further, at the lower transition temperature $T_{c1}$, the system exhibits a reentrant transition into the normal phase, in which quantum fluctuations destroy both algebraic order and long-range order, leading to finite resistivity.

If $\alpha$ is increased, $K_1$ and $K_2$ approach each other, implying that the superconducting order parameter shrinks. Finally, when $\alpha$ becomes equal to $\alpha_c$, $K_1$ and $K_2$ coalesce; beyond this value of $\alpha$, the entire spin-wave fixed line is unstable to vortices. This means that quantum fluctuations are strong enough to destroy superconductivity at all temperatures.

However, the detailed prediction of Eqs. (3.5) is in fact untrustworthy because the variational method, like the perturbation expansion, is semiclassical in nature and is expected to be accurate only in the classical region $K \alpha \ll 1$. Note that $K \alpha = \theta u / 12$ is a measure of the relevance of the charging energy with respect to thermal fluctuations. For example, it is obvious that Eqs. (3.5) do not give a correct description near zero temperature, where $K \alpha \gg 1$. In particular, at zero temperature Eqs. (3.5) incorrectly predict the existence of the normal phase for all nonzero values of $\alpha$. Rigorous analysis, on the other hand, indicates the existence of a nonzero $\alpha$ below which the system is superconducting, since the system at zero temperature should behave like a three-dimensional $XY$ model with $\alpha$ taking the role of the temperature. Therefore, we cannot ascertain whether the predicted reentrant transition is generic to the system considered in this paper or is just an artifact of the variational method, although the variational method usually gives fairly reasonable results even at rather low temperatures.

IV. CONCLUDING REMARKS

In this paper, we have investigated the effects of fluctuations on the critical behavior of 2D periodic arrays of coupled Josephson junctions. Both random fluctuations of the external magnetic field and quantum fluctuations have been considered.

Interpreting fluctuations of the magnetic field having a Gaussian distribution in terms of a random frustration, we use the replica method to analyze this randomly frustrated system. It has been suggested that for some simple (mean) values of the frustration, this system may be decomposed into coupled $XY$ models with lattice anisotropy. In this way it has been shown that weak fluctuations are irrelevant, whereas strong fluctuations destroy the superconducting phase. For medium fluctuations it has been argued that their effects depend strongly on the lattice structure. On a square lattice they are irrelevant, while on a triangular lattice they lead to unfrustrated $XY$ models.

We next have used the variational method to study the effects of quantum fluctuations. Within this semiclassical approximation the Josephson coupling between neighbors is shown to be renormalized in a significant way by quantum fluctuations. This effect on the corresponding RG equations has been analyzed to show the existence of a low-temperature reentrant transition into the normal phase. Also shown is that sufficiently strong quantum fluctuations can destroy superconductivity at all temperatures.

Since both fluctuations of the magnetic field and quantum fluctuations are unavoidable in real experiments, the problems considered in this paper are quite relevant to experiments. However, the analysis presented in this paper may not be of direct relevance to real experiments because of several simplifications. First, we have taken the thermodynamic limit, which is obviously not the case for real experiments using artificially structured systems such as Josephson-junction arrays. Especially, the existence of transitions according to values of the frustration would become ambiguous and the idea of the effects of field fluctuations would not have much meaning. Moreover, we have neglected both the temperature dependence of the Josephson coupling and the off-diagonal charging energy. The latter should be taken into account for systems which are not very densely packed. These complications may modify the effects of quantum fluctuations in real experiments. Above all, the most uncertainty obviously comes from the semiclassical nature of the variational method used to investigate the effects of quantum fluctua-
tions. This prevents us from ascertaining the existence of a reentrant transition even in the ideal system considered in this paper. Recently this reentrant behavior has been observed in experiments on some thin films. However, this may not be the result of quantum fluctuations considered in this paper, since other effects are likely to play significant roles in those thin films. Perhaps extensive Monte Carlo simulations for frustrated systems, where the commensurate-incommensurate effect and the finite-size effect in time as well as in space are carefully considered, may confirm the results obtained in this paper.

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21This result is naturally in agreement with that of the perturbation expansion up to the first order in βu. However, they do not agree in higher-order corrections. The variational method in general gives better results than the perturbation expansion. See Ref. 19.