

Superfluid inhomogeneity and microwave absorption in a model for thin high- T_c superconducting films

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Abstract

We investigate the microwave absorption arising from inhomogeneity in the superfluid density of thin high- T_c superconducting films. Such inhomogeneities may arise from a wide variety of sources, including quenched random disorder and static charge density waves such as stripes. We show that both mechanisms will inevitably produce additional absorption at finite frequencies. We present simple model calculations for this extra absorption, and discuss applications to other transport properties in high- T_c materials. Finally, we discuss the connection of these predictions to recent measurements by Corson et al. (Nature (London) 398 (1999) 221) of absorption by the high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ in the THz frequency regime.

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1. Introduction

Thin films of the high- T_c superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ show a remarkably strong absorption in the microwave regime, even far below T_c [1,2]. In conventional, low- T_c , s-wave superconductors, there is no such background, because there is no absorption below the energy gap for pair excitations, 2Δ . But in high- T_c materials, which are thought to have a $d_{x^2-y^2}$ order parameter, the gap vanishes in certain nodal k directions. Hence, gapless nodal

quasiparticles can be excited, and hence can absorb microwave radiation, at arbitrarily low temperatures. However, experiment suggests that this absorption is much stronger than expected from the quasiparticles alone [3].

In this paper, we show that such extra absorption can be produced if the superfluid density $n(\mathbf{x})$ within the ab plane is a function of position \mathbf{x} . Such spatial variation can be produced, e.g., by quenched disorder, or by charge density waves (e.g., stripes). The extra absorption is most likely in the microwave frequency range. A similar absorption was found by us in a previous paper [4], in which the inhomogeneity was described as static fluctuations in the Josephson coupling between superconducting grains.

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2. Microwave absorption in an inhomogeneous two-dimensional superfluid

We consider a two-fluid model of a superconductor in two dimensions, with local conductivity

$$\sigma(\mathbf{x}, \omega) = \sigma_{\text{qp}}(\omega) + \frac{iq^2 n_s(\omega, \mathbf{x})}{m^* \omega}, \quad (1)$$

where $q = 2|e|$ and m^* is twice the electron mass m_e . $n_s(\mathbf{x}, \omega)$ is assumed spatially varying, while σ_{qp} is taken to be spatially uniform. We wish to calculate the complex effective conductivity $\sigma_e(\omega)$. σ_{qp} might be a contribution from the nodal quasiparticles, while n_s represents the perfect-conductivity response of the superconductor. It can be spatially varying because of the very short in-plane coherence length.

We employ the Kramers–Kronig relations satisfied by $\sigma_e(\omega)$, namely

$$\sigma_{e1}(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\omega' \sigma_{e2}(\omega')}{\omega'^2 - \omega^2} d\omega' + \sigma_\infty$$

$$\text{and } \sigma_{e2} = \frac{q^2 n_{s,e}}{m^* \omega} - \frac{2\omega}{\pi} P \int_0^\infty \frac{\sigma_{e1}(\omega') - \sigma_\infty}{\omega'^2 - \omega^2} d\omega',$$

where P means “principal part of,” $\sigma_e = \sigma_{e1} + i\sigma_{e2}$, and $n_{s,e}$ is the effective superfluid density [the value of $q^2 n_{s,e}/m^*$ can be found from the residue of $\sigma_e(\omega)$ at $\omega = 0$]. At very large frequencies ω , the equation for σ_{e2} becomes

$$\sigma_{e2} \rightarrow \frac{q^2 n_{s,e}}{m^* \omega} + \frac{2}{\pi \omega} \int_0^\infty [\sigma_{e1}(\omega') - \sigma_\infty] d\omega'.$$

2.1. Frequency-independent σ_{qp}

If $\sigma_{\text{qp}}(\omega)$ is real and frequency independent, then $\sigma_\infty = \sigma_{\text{qp}}$. Then at high frequencies, the local complex conductivity, $\sigma_\infty + iq^2 n_s(\mathbf{x})/(m^* \omega)$, has only small spatial fluctuations. Then (see, e.g., Ref. [5])

$$\sigma_e \approx \sigma_{\text{av}} - \frac{1}{2} \frac{\langle (\delta\sigma)^2 \rangle}{\sigma_{\text{av}}}, \quad (2)$$

$\delta\sigma(\omega, \mathbf{x}) \equiv \sigma(\omega, \mathbf{x}) - \sigma_{\text{av}}(\omega)$, where $\sigma_{\text{av}}(\omega) = \sigma_\infty + iq^2 n_{s,\text{av}}/(m^* \omega)$, $\delta\sigma(\omega, \mathbf{x}) = \sigma(\omega, \mathbf{x}) - \sigma_{\text{av}}(\omega)$, and $\langle \dots \rangle$ is a space average. Since only n_s , and not

σ_{qp} , is fluctuating, this expression simplifies to

$$\sigma_e \approx \sigma_\infty + \frac{iq^2 n_{s,\text{av}}}{m^* \omega} + \frac{1}{2} \frac{(q^2/m^*)^2 \langle (\delta n_s)^2 \rangle}{\omega^2 \sigma_\infty + iq^2 \omega n_{s,\text{av}}/m^*}. \quad (3)$$

At large ω , the imaginary part of σ_e to leading order in $1/\omega$ is simply $\sigma_{e2} \sim q^2 n_{s,\text{av}}/(m^* \omega)$. Equating this expression to the right-hand side of the Kramers–Kronig expression for σ_{e2} , we finally obtain

$$\int_0^\infty [\sigma_{e1}(\omega') - \sigma_\infty] d\omega' = \frac{q^2 \pi}{2m^*} (n_{s,\text{av}} - n_{s,e}). \quad (4)$$

If $n_s(\mathbf{x})$ is spatially varying, the right-hand side is always positive, whence *there will be an additional contribution to $\sigma_{e1}(\omega)$, beyond σ_{qp}* .

At small ω , Eq. (3) implies

$$\sigma_{e2} \sim \frac{q^2 n_{s,\text{av}}}{m^* \omega} + \frac{1}{2} \left(\frac{q^2/m^* \langle (\delta n_s)^2 \rangle}{\omega n_{s,\text{av}}} \right),$$

and thus

$$n_{s,\text{av}} - n_{s,e} \approx \frac{q^2 n_{s,\text{av}} \langle (\delta n_s)^2 \rangle}{m^*} / (2n_{s,\text{av}}^2).$$

Then Eq. (4) becomes

$$\int_0^\infty [\sigma_{e1}(\omega') - \sigma_\infty] d\omega' \approx \frac{q^2 \pi n_{s,\text{av}} \langle (\delta n_s)^2 \rangle}{4m^* n_{s,\text{av}}^2}. \quad (5)$$

For fixed $\langle (\delta n_s)^2 \rangle / n_{s,\text{av}}^2$, this integral is proportional to $n_{s,\text{av}}$. That is, the extra integrated fluctuation contribution to σ_{e1} , is proportional to the average superfluid density. A similar result has been reported in experiments [1,3].

2.2. Drude $\sigma_{\text{qp}}(\omega)$

For a Drude $\sigma_{\text{qp}}(\omega) = \sigma_0/(1 - i\omega\tau)$, we can carry out a similar analysis [6]. In the case of weak fluctuations in n_s , the result is [6]

$$\begin{aligned} & \int_0^\infty [\sigma_{e,1}(\omega') - \sigma_{\text{qp},1}(\omega')] d\omega' \\ & \approx \frac{\pi}{4} ((q^2/m^*)^2 \langle (\delta n_s)^2 \rangle) \\ & \quad \times \left(\frac{1}{q^2 n_{s,\text{av}}/m^*} - \frac{1}{q^2 n_{s,\text{av}}/m^* + \sigma_0/\tau} \right). \end{aligned} \quad (6)$$

In the limit $\sigma_0/\tau \gg q^2 n_{s,\text{av}}/m^*$, this expression reduces to the right-hand side of Eq. (5). Thus, in this regime, the extra spectral weight is indeed proportional to the average superfluid density $n_{s,\text{av}}$.

2.3. Tensor n_s

Next, we consider a *tensor* superfluid density, as would be expected in a superconducting layer containing quenched charge density waves such as a charge stripes [7]. In this case, the (2×2) superfluid density tensor should have the form $n_s^{2\beta}(\mathbf{x}) = R^{-1}(\mathbf{x})n_s^d R(\mathbf{x})$, where n_s^d is a diagonal 2×2 matrix with diagonal components $n_{s,A}$, $n_{s,B}$, and $R(\mathbf{x})$ is a position-dependent 2×2 rotation matrix. Describing the relative orientation of the charge density wave or stripes. If the stripes have either of two orientations along the crystal axes of the layer, with equal probability, then the *effective* superfluid density $n_{s,e}$ will be a scalar.

The arguments of the previous subsections can readily be transferred to the tensor case. For a two-fluid model with a frequency-independent quasiparticle conductivity which has the same value σ_∞ in both the A and B directions, the effective scalar conductivity $\sigma_e(\omega)$ again satisfies Kramers–Kronig relations, and one again obtains the sum rule (4), where $n_{s,av}$ is now the rotational average of a diagonal element of $n_{s;\alpha\beta}$. If the principal axes of $n_{s;\alpha\beta}$ point with equal probability along the two symmetry directions of the CuO_2 plane, as mentioned above, $n_{s,av} = (n_{s,A} + n_{s,B})/2$. Likewise, if the principal axes were to point in any direction in the plane with equal probability (a circumstance which seems unlikely for a stripe phase), then it can be shown [8] that once again $n_{s,av} = (n_{s,A} + n_{s,B})/2$.

If the quasiparticle conductivity is frequency dependent, then the analogous scalar results of the previous section continue to hold. For example, if $\sigma_{qp,A}(\omega) = \sigma_{qp,B}(\omega)$, then the extra spectral weight due to the superfluid inhomogeneity is again given by Eq. (5) in the weak-inhomogeneity regime.

In the case of the stripe geometry, where the principal axes of the conductivity tensor take either of two perpendicular orientations with equal probability, $n_{s,e}$ is given by the duality result [5,7]

$$n_{s,e} = \sqrt{n_{s,A}n_{s,B}}. \quad (7)$$

This form allows the extra spectral weight to be evaluated straightforwardly, given $\sigma_{qp}(\omega)$, without making the small fluctuation approximation.

3. Numerical example

As a simple example, we consider a superconducting layer in which the conductivity has one of two possible values, $\sigma_A(\omega)$ or $\sigma_B(\omega)$ with equal probability, and we assume $\sigma_{A,B}(\omega) = \sigma_{qp}(\omega) + q^2 n_{s;A,B}/m^* \omega$, where we take $n_{s,A} > n_{s,B}$. This model would be suitable either for a layer with static scalar disorder, or for a model of stripe domains, as discussed above. We also write $\sigma_{qp} = \sigma_0/(1 - i\omega\tau_{qp})$, with $\sigma_0 = n_{qp}q^2\tau_{qp}/m^*$. We also assume $n_{qp} = \alpha T$ for $T < T_c$ or $n_{qp} = \alpha T_c$ for $T > T_c$, $\tau_{qp}^{-1} = \beta T$, $n_{s,A} = \gamma n_{s,0}$, $n_{s,B} = \gamma^{-1} n_{s,0}$, where $n_{s,0}(T) = n_{s,0}(0)\sqrt{1 - 2\alpha T/n_{s,0}(0)}$, and γ is a parameter describing the superfluid inhomogeneity (this form ensures that $n_{s,av} \equiv \sqrt{n_{s,A}n_{s,B}} = n_{s,0}$). Our choice of temperature dependence for $n_{s,0}(T)$ ensures that $n_{s,0}$ (i) decreases linearly with increasing temperature T at small T , as observed experimentally [10]; and (ii) vanishes at a critical temperature T_c as $\sqrt{T_c - T}$. The ingredients of this model are very similar to those of Ref. [3], and have a straightforward interpretation. First, $n_{s,qp}$ should be proportional to T in a gapless d-wave superconductor [11], and hence $n_{s,e}$ should be depleted by the same amount and fall off linearly in T . We also assume that $n_{s,A}$ and $n_{s,B}$ individually are linear in T . The form $1/\tau_{qp} = \beta T$, where β is another constant [3], has been observed for nodal quasiparticles in the superconducting state [12].

We compute σ_e using Bruggeman effective-medium approximation (EMA) [5,9], which gives $(\sigma_A - \sigma_e)/(\sigma_A + \sigma_e) + (\sigma_B - \sigma_e)/(\sigma_B + \sigma_e) = 0$. The solution to this equation is simply $\sigma_e = \sqrt{\sigma_A\sigma_B}$. Fig. 1 shows the resulting $\sigma_{e,1}(\omega, T)$ for several frequencies ranging from 0.2 to 0.8 THz, the range measured in Ref. [3]. The parameters T_c , σ_0 , $n_{s,0}(0)$, α and β were taken from Ref. [3], and we assumed $\gamma = 3$. Also shown are $\int_{\omega_{\min}}^{\omega_{\max}} \sigma_{e,1}(\omega, T) d\omega$ for $\omega_{\min}/(2\pi) = 0.2$ THz and $\omega_{\max}/(2\pi) = 0.8$ THz. Finally, we plot $\sigma_{qp,1}(\omega, T)$ for these frequencies, as well as $\int_{\omega_{\min}}^{\omega_{\max}} \sigma_{qp,1} d\omega$. Clearly, $\sigma_{e,1}(\omega)$ is considerably increased beyond the quasiparticle contribution, because of spatial fluctuations in the superfluid density.

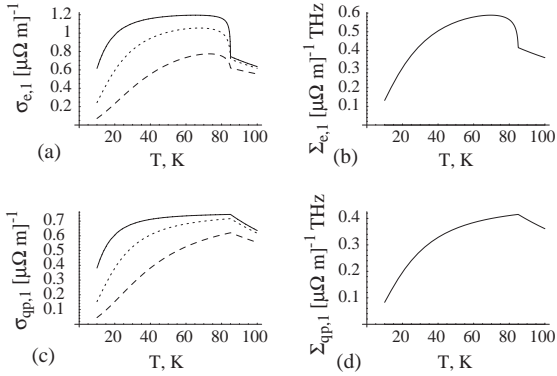


Fig. 1. (a) $\sigma_{e,1}(\omega, T)$, for $\omega/(2\pi) = 0.2$ THz (solid line), 0.4 THz (dotted line), and 0.8 THz (dashed line), for the model inhomogeneous superconductor described in the text. Also plotted are (b) $\Sigma_{e,1} \equiv \int_{\omega_{\min}}^{\omega_{\max}} \sigma_{e,1}(\omega) d\omega$, (c) $\sigma_{qp,1}(\omega, T)$, and (d) $\Sigma_{qp,1} \equiv \int_{\omega_{\min}}^{\omega_{\max}} \sigma_{qp,1}(\omega) d\omega$, where $\omega_{\min}/(2\pi) = 0.2$ THz and $\omega_{\max}/(2\pi) = 0.8$ THz.

4. Discussion

We have shown that a superconducting layer with an inhomogeneous superfluid density will have an extra absorption not present in a homogeneous superconductor. The frequency integral of $\sigma_{e,1}(\omega)$ associated with this absorption is proportional to the superfluid density, in general agreement with experiments [1,3]. We have also shown that this inhomogeneity, and hence the extra absorption seen in experiments, can arise from a stripe domain structure, as well as from random but isotropic disorder. Thus, we speculate

that no such extra absorption should be observed in a high- T_c superconductor unless one of these two types of inhomogeneities are present (beyond that expected purely from nodal quasiparticle absorption).

Acknowledgements

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