

Negative magnetoresistance produced by Hall fluctuations in a ferromagnetic domain structure

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We present a model for a negative magnetoresistance (MR) that would develop in a material with many ferromagnetic domains even if the individual domains have *no* magnetoresistance and even if there is no boundary resistance. The negative MR is due to a classical current-distortion effect arising from spatial variations in the Hall conductivity, combined with a change in domain structure due to an applied magnetic field. The negative MR can exceed 1000% if the product of the carrier relaxation time and the internal magnetic field due to spontaneous magnetization is sufficiently large. © 2001 American Institute of Physics. [DOI: 10.1063/1.1392978]

There has recently been much interest in materials with a *negative* magnetoresistance (MR). In such materials, the resistivity in a particular direction decreases when a magnetic field is applied. For example, the doped manganites show a very large decrease known as colossal magnetoresistance.¹⁻³ Similar behavior is achieved at comparatively small fields in suitably strained manganite films.^{4,5} While the mechanism of these effects is not fully understood, it may arise from spin-dependent scattering at interfaces between domains, or within an individual domain.

In this note, we discuss a model which could produce a negative MR in a multidomain material even without any spin-dependent or interfacial scattering. This negative MR arises from what are sometimes called “path length effects” (see, e.g., Ref. 6). The model is based on only two assumptions: (i) the medium is macroscopically inhomogeneous, with a spatially varying Hall conductivity; and (ii) the inhomogeneous domain structure depends on the applied magnetic field in a suitable way. Given (i) and (ii), one may extract the effective resistivity tensor ρ_e of the medium as a function of field using standard methods as discussed, for example, in Ref. 7. We will show that a simple, plausible model for the conductivity tensors of the individual domains, combined with a suitable method for calculating ρ_e , gives rise to a negative MR which can be very substantial.

We assume that there are two types of domains (two components), with magnetization parallel or antiparallel to the z axis. The two components have volume fractions of p and $1-p$, respectively. We carry out our calculation for two types of domain geometry.⁸⁻¹⁰ In the first geometry (a “random columnar” microstructure), the domains have columnar symmetry with the columnar axis also lying along the z axis [see Fig. 1(a)]. This microstructure could represent a film with an out-of-plane easy magnetization axis and a random distribution of domains. The second geometry consists of a collection of parallel slabs arranged perpendicular to the x axis and infinite in the y direction [see Fig. 1(b)]. This geometry models domain structures obtained when the material is demagnetized in a certain way (see, e.g., Ref. 1).

For both geometries, we assume that the ferromagnetic metal has a free-electron conductivity tensor with an effective magnetic field generated *internally*, by the magnetization. Thus, we assume that the internal field is $\pm B\hat{z}$ in the two components, where \hat{z} is a unit vector in the direction normal to the plane. The 3×3 conductivity matrices for the two components have elements

$$\sigma_{1,xx} = \sigma_{1,yy} = \sigma_{2,xx} = \sigma_{2,yy} = \sigma_0 / [1 + h^2], \quad (1)$$

$$\sigma_{1,xy} = -\sigma_{1,yx} = -\sigma_{2,xy} = \sigma_{2,yx} = \sigma_0 h / [1 + h^2], \quad (2)$$

$$\sigma_{1,zz} = \sigma_{2,zz} = \sigma_0, \quad (3)$$

with all other elements of σ_1 and σ_2 equal to zero. Here, $h = \omega_c \tau$ is a dimensionless magnetic field, $\omega_c = qB/(m^*c)$ is the cyclotron frequency associated with the internal field B , and τ is a suitable relaxation time.

We will calculate the effective conductivity tensor in both geometries using the standard effective medium approximation.⁷ (In the case of the parallel slabs geometry, this approximation is, in fact, exact.⁷) In both geometries, the self-consistency condition takes the form

$$p \delta\sigma_1 [1 - \Gamma \delta\sigma_1]^{-1} + (1-p) \delta\sigma_2 [1 - \Gamma \delta\sigma_2]^{-1} = 0. \quad (4)$$

Here, $\delta\sigma_i = \sigma_i - \sigma_e$, where σ_i is the conductivity tensor of the i th component and σ_e is the effective (3×3) conductivity tensor; and Γ is the depolarization tensor,⁷ which is different in the two geometries.

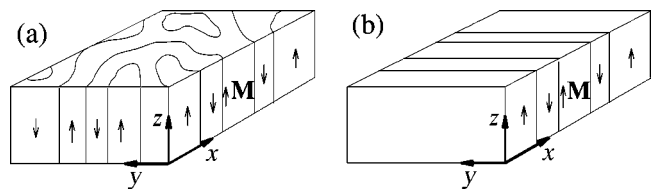


FIG. 1. Schematic of the discussed geometries. (a) “Random columnar” microstructure, consisting of columnar domains with oppositely directed magnetizations \mathbf{M} and internal magnetic inductions \mathbf{B} parallel to the columnar axes. (b) “Parallel slabs” microstructure, with directions of \mathbf{M} and internal field \mathbf{B} indicated by arrows. The domains are separated by parallel domain walls. In both cases, \mathbf{B} is assumed uniform within a domain and parallel to the domain walls, and, in both cases, a thin film can be viewed as a slice of the microstructure, as shown.

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In the random columnar geometry Γ is diagonal with diagonal elements $\Gamma_{xx}=\Gamma_{yy}=-1/[2\sigma_{e,xx}]$; $\Gamma_{zz}=0$, where we have used the fact that σ_e is antisymmetric and that $\sigma_{e,xx}=\sigma_{e,yy}$. The self-consistency condition (4) can readily be multiplied out, with the result

$$p\delta\sigma_1+(1-p)\delta\sigma_2=\delta\sigma_2\Gamma\delta\sigma_1. \quad (5)$$

This is a matrix equation for the effective conductivity tensor σ_e , whose components enter in Eq. (5) via both $\delta\sigma_i$ and Γ . For our choice of conductivities, the solution to this equation is

$$\sigma_{e,xx}=\sigma_{e,yy}=\frac{\sigma_{1,xx}}{\sqrt{1-4p(1-p)h^2/[1+h^2]}}, \quad (6)$$

$$\sigma_{e,xy}=-\sigma_{e,yx}=\frac{(2p-1)\sigma_{1,xy}}{\sqrt{1-4p(1-p)h^2/[1+h^2]}}, \quad (7)$$

$$\sigma_{e,zz}=\sigma_{1,zz}, \quad (8)$$

with other components vanishing.

We first consider the special case $p=1/2$. Then Eq. (7) reduces to $\sigma_{e,xy}=0$, while Eq. (6) is equivalent to

$$\sigma_{e,xx}=\sqrt{\sigma_{1,xx}^2+\sigma_{1,xy}^2}. \quad (9)$$

The in-plane resistivity is $\rho_{e,xx}=[\sigma_e^{-1}]_{xx}=1/\sigma_{e,xx}$, where the last condition follows from the fact that $\sigma_{e,xy}=-\sigma_{e,yx}=0$. Using our particular forms for $\sigma_{1,xx}$ and $\sigma_{1,xy}$, and using Eq. (9), we finally get

$$\rho_{e,xx}\left(p=\frac{1}{2}\right)=\frac{\sqrt{1+h^2}}{\sigma_0}. \quad (10)$$

For comparison, we can calculate the resistance for the case $p=1$ (or $p=0$), which corresponds to a homogeneous magnetic material. Since we have assumed a free-carrier conductivity tensor, the resistance is simply

$$\rho_{e,xx}(p=1)=\sigma_0^{-1}. \quad (11)$$

Thus, if an applied field causes all the domains to line up parallel with the field, the resistivity $\rho_{e,xx}$ will be reduced, i.e., there will be a *negative* MR.

Next, we consider the parallel slabs microgeometry. In this case, the resistivity difference between the $p=1/2$ and $p=1$ cases may be even larger. For this geometry, the only nonzero element of the depolarization tensor Γ is $\Gamma_{xx}=-1/\sigma_{e,xx}$.¹¹ Carrying out the algebra in the self-consistency condition (4), we find that all the elements of $\rho_e\equiv(\sigma_e)^{-1}$ vanish except

$$\rho_{e,xx}=\sigma_0^{-1}[1+h^2-(2p-1)^2h^2], \quad (12)$$

$$\rho_{e,xy}=-\rho_{e,yx}=-\frac{(2p-1)h}{\sigma_0}, \quad (13)$$

$$\rho_{e,yy}=\rho_{e,zz}=\frac{1}{\sigma_0}. \quad (14)$$

In particular, at $p=1/2$

$$\rho_{e,xx}=\frac{1+h^2}{\sigma_0}, \quad (15)$$

whereas at $p=1$ $\rho_{e,xx}$ is still given by Eq. (11). Note that in the columnar microgeometry $\rho_{e,xx}$ is the resistivity perpendicular to the columns (or parallel to the film), while in the parallel slabs microgeometry, $\rho_{e,xx}$ represents the resistivity perpendicular to the slab surfaces.

Our picture of the negative MR in these structures is now the following. At zero (or very low) applied magnetic field, the sample has either a random columnar or a slab microgeometry, with approximately equal numbers of up and down domains. The resistivity then corresponds to $p=1/2$. As the applied field is increased, the domains parallel to the field grow at the expense of the antiparallel domains, so that eventually the resistivity is close to that for $p=1$. The total fractional change in ρ_{xx} (with respect to the low resistance value) is just

$$\frac{\Delta\rho}{\rho}\equiv\frac{\rho(p=1)-\rho(p=1/2)}{\rho(p=1)}=1-\sqrt{1+h^2}<0, \quad (16)$$

for the random columnar domain structure; and

$$\frac{\Delta\rho}{\rho}=-h^2, \quad (17)$$

for the parallel slabs domain structure. Both of these correspond to a *decrease* in resistivity.

To compare these analytic results to experiments, the quantity $2p-1$ must be related to the external magnetic field \mathbf{H} . Typically, in a sample with domains, the average sample magnetization \mathbf{M}_e is a hysteretic function of \mathbf{H} . As an illustration, we model this function by $M_e=M_{\text{sat}}\tanh((H\pm H_{\text{cr}})/H_{\text{sat}})$, where H_{cr} is the coercive field, H_{sat} and M_{sat} are characteristic saturation values of the external field and magnetization, and the sign choice depends on the direction in which the hysteresis loop is traversed. If the *local* magnetization can have only the two values $\pm M_{\text{sat}}$ in the up and down domains, then $M_e=M_{\text{sat}}(2p-1)$, and, hence,

$$2p-1=\tanh\left(\frac{H\pm H_{\text{cr}}}{H_{\text{sat}}}\right). \quad (18)$$

We have used this expression in the above formulas for $\rho_{e,xx}$ to obtain the hysteresis in $\rho_{xx}(H)$; the results are shown in Fig. 2. Evidently both the shape and the positions of the peaks in $\rho_{e,xx}(H)$ depend on the domain microgeometry (i.e., whether it is “random columnar” or “parallel slabs”) and the squareness of the hysteresis loop (controlled by the parameter $H_{\text{cr}}/H_{\text{sat}}$).

The *magnitude* of $\Delta\rho_{xx}/\rho_{xx}$ is entirely controlled by the parameter $h=\omega_c\tau$ [cf. Eqs. (16) and (17)]. At room temperature,¹² h is likely to be small: assuming $\tau=2\times 10^{-14}$ s at $T=300$ K (1/10 that of Cu), a local magnetization $M\sim 10^3$ G and internal $B\sim 4\pi M$, we find $\omega_c\tau\sim 10^{-3}$. But at low temperatures, h could be very large: again assuming $\tau\sim 0.1\tau_{\text{Cu}}$ at $T=4$ K, we find $\omega_c\tau\sim 10^2$, implying a very large negative $\Delta\rho_{xx}$, especially in the slab geometry. In reality both B and τ are likely to be reduced near the domain walls,¹³ nevertheless, the change in ρ_{xx} could easily exceed 1000%, according to Eqs. (16) and (17).

Remarkably, these results resemble experimental observations in strained manganite films.⁴ In these films, the magnetoresistance is large and negative at low T , but is negligible above 100 K. Moreover, our prediction (see Fig. 2),

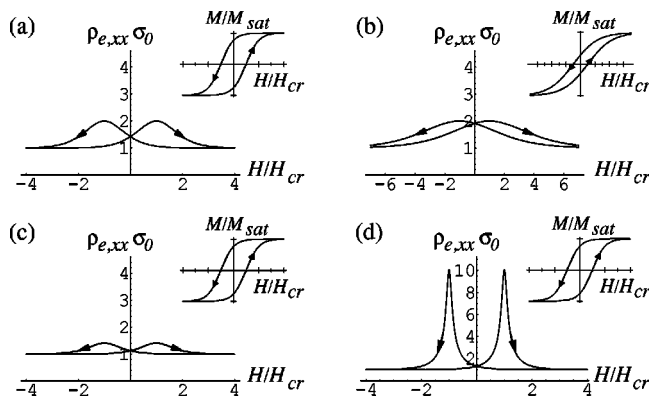


FIG. 2. Predicted behavior of the effective resistivity $\rho_{e,xx}$, plotted vs applied magnetic field \mathbf{H} . (a) and (b): parallel slab microgeometry, [Eqs. (6)–(7)]; (c) and (d): random columnar microgeometry [Eqs. (12)–(14)]. The plots are shown for several different values of the parameters H_{cr}/H_{sat} and $\omega_c\tau$, using Eq. (18): (a) and (c) $H_{cr}/H_{sat}=1$, $\omega_c\tau=1$; and (b) $H_{cr}/H_{sat}=0.3$, $\omega_c\tau=1$; and (d) $H_{cr}/H_{sat}=1$, $\omega_c\tau=10$. Insets show corresponding magnetization hysteresis loops, also obtained using Eq. (18).

that the magnetic field of the peak resistance is close to the coercive field, is also consistent with experiment. Of course, there are certainly other, probably more significant, mechanisms contributing to the negative MR in manganite films (such as colossal magnetoresistance or spin-dependent scattering). Nonetheless, it seems plausible that this Hall-induced negative MR may play a significant role in some materials.

In conclusion, we have demonstrated that in a ferromagnet with a suitable domain structure, the Hall effect arising from the local magnetic field will increase the effective resistivity of the material. When an external magnetic field destroys the domain structure, it will also reduce the resistivity, thereby generating a substantial negative MR. The magnitude of this effect is controlled by the parameter $\omega_c\tau$, which may be very large at low temperatures, and by details

of the domain microgeometry, as well as by the shape of the hysteresis curve $M(H)$.

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- ¹R. von Helmolt, J. Wecker, B. Holzapfel, L. Schultz, and K. Samwer, Phys. Rev. Lett. **71**, 2331 (1993).
- ²K. Chahara, T. Ohno, M. Kasai, and Y. Kozono, Appl. Phys. Lett. **63**, 1990 (1993).
- ³S. Jin, T. H. Tiefel, M. McCormack, R. A. Fastnacht, R. Ramesh, and L. H. Chen, Science **264**, 413 (1994).
- ⁴H. S. Wang and Q. Li, Appl. Phys. Lett. **73**, 2360 (1998); H. S. Wang, Q. Li, K. Liu, and C. L. Chien, *ibid.* **74**, 2212 (1999).
- ⁵Y. Wu, Y. Suzuki, U. Rüdiger, J. Yu, A. D. Kent, T. K. Nath, and C. B. Eom, Appl. Phys. Lett. **75**, 2295 (1999).
- ⁶J. O'Donnell, J. N. Eckstein, and M. S. Rzechowski, Appl. Phys. Lett. **76**, 218 (2000).
- ⁷D. J. Bergman and D. Stroud, Solid State Phys. **46**, 147 (1992); the effective resistivity tensor ρ_e relates the space-averaged effective field and current density, as defined in that paper.
- ⁸We assume that the magnetization is normal to the film surface. In the opposite case of in-plane magnetization, the results for the slab geometry would still be exact, provided that the domain diameter d is smaller than the film thickness t . The latter condition will probably not be satisfied in very thin films, as one expects $d \sim \sqrt{t}$ (see Ref. 9). When $d \gg t$ and the magnetization is in the plane of the film (as, e.g., in Ref. 10), the effective film resistivity $\rho_{e,xx}$ will depend on the number and positions of the domain walls, rather than on the volume fractions p and $1-p$ of the domains as it does in our model.
- ⁹L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, (Pergamon, Oxford, NY, 1984).
- ¹⁰N. D. Mathur, P. B. Littlewood, N. K. Todd, S. P. Isaac, B.-S. Teo, D.-J. Kang, E. J. Tarte, Z. H. Barber, J. E. Evetts, and M. G. Blamire, J. Appl. Phys. **86**, 6287 (1999).
- ¹¹For scalar conductivities, these values would lead to the familiar result that resistivities add in the direction perpendicular to the slab interfaces, while conductivities add in the parallel directions.
- ¹²For experimental results at room temperatures, see, e.g., R. A. Hajjar and M. Mansuripur, J. Appl. Phys. **72**, 1528 (1992).
- ¹³This negative MR would probably be little affected even if \mathbf{B} varies smoothly in direction without changing its magnitude close to a domain wall, as likely in a real material.