Physics 880A20 COSMOLOGY Problem Set 9 November 20, 2006

Note: The numbers in [ ] are the point totals for each part of each problem.

Problem 1 :

Suppose (as was once claimed) that the tau-neutrino has a mass of 17 keV and is stable or long-lived ($\tau_{\nu_\tau} \gg t_{\rm BBN}$). Assume this heavy tau-neutrino has the usual standard-model weak interactions so that the ν_τ decoupled at a temperature of ~ 3 MeV.

a) If BBN "starts" (the D-bottleneck is broken) when the temperature is 70 keV, find the ratio of the total energy density to the photon energy density and the age of the Universe at this time (you may assume that $e^{\pm}$ annihilation is complete by this time). [10]

b) Find the temperature (indicate whether it is the photon or neutrino temperature) when the ν_τ begin to dominate the total energy density of the Universe. [10]

c) If the ν_τ were stable, find Ω_{ν_τ}, the ratio of the present energy density in ν_τ to the present critical density (you may assume $\mathrm{H}_0 = 70$ km/s/Mpc). [10]

Problem 2 :

Suppose the early Universe were filled with a homogeneous magnetic field of magnitude B. Flux conservation implies that as the Universe expands the magnetic field strength decreases as $B \propto a^{-2}$. Note: for a magnetic field B measured in Tesla, the energy density in eV/cm^3 is $\rho_B = 2.5 \times 10^{18} B^2$; $1\mu\mathrm{G} = 10^{-10}\mathrm{T}$.

a) How large would this field have to be at $T_\gamma = 1$ MeV to have a significant effect on primordial nucleosynthesis? [5]

b) How would the presence of such a field (which saturates your limit in part a) affect the predicted abundance of helium-4? (Even though the magnetic field could have a small effect on the $n \leftrightarrow p$ weak rates, neglect this possibility here.) [5]

c) If the magnetic field had the value you found in part a), what would its field strength and energy density (as a fraction of the criticial density) be today? [5]

Problem 3 :

In the standard models of particle physics and cosmology all three flavors of neutrinos are kept in equilibrium via the Z^0-mediated neutral current interactions: $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$. Ignore here the charged-current contributions which affect only the ν_e. The thermally averaged rate for these interactions scales as $\langle \sigma v \rangle \propto T^2/M_Z^4$. In the standard model the neutrino-decoupling temperature is $\mathrm{T}_{\nu d} \approx 3$ MeV. Suppose we lived in a Universe in which the Z boson were more massive: $M_{Z'}/M_Z = 15$.

a) Find the decoupling temperature of the neutrinos in this Universe. [10]

b) Evaluate the energy density at 1 MeV and compare it to what it is in "our" Universe. That is, at 1 MeV, find: g_{eff}, ΔN_ν, and the speed-up factor S. If all other physics were unchanged, would more or less helium-4 have been produced during BBN in this Universe? [15]

c) If the present CBR temperature is $T_{\gamma 0} = 2.725$ K, find $T_{\nu 0}$, the present neutrino temperature in this Universe. [5]

HAPPY THANKSGIVING!