

Note: The numbers in [] are the point totals for each part of each problem.

Problem 1 :

In the early Universe the charged-current weak interactions keep neutrons and protons in equilibrium (“chemical” equilibrium) until (approximately) the weak rate (Γ_{wk}) becomes slower than the expansion rate (H). For the weak rate assume that $\Gamma_{wk} = 2.34T_{\text{MeV}}^5 \text{ s}^{-1}$.

- a) Find the “freeze-out” temperature (a misnomer!), T_f , at which the weak rate is equal to the expansion rate. [5]
- b) Assuming that the n/p ratio was in equilibrium at $T = T_f$, find the neutron to proton ratio at “freeze-out”. (Note that neutrons are heavier than protons by 1.29 MeV.) [5]

Problem 2 :

Suppose there is a new, very weakly interacting, “sterile” neutrino ν_s which does **not** mix with the usual (“active”) neutrinos. The ν_s are light but not massless ($0 \neq m_{\nu_s} \ll 1 \text{ MeV}$).

- a) If these sterile neutrinos decouple at a temperature of 10 GeV, find their contribution to $\Delta N_\nu \equiv \rho_{\nu_s}/\rho_\nu$, the effective number of equivalent neutrinos at $T_\gamma \approx 1 \text{ MeV}$. [10]
- b) Assume that these ν_s are non-relativistic at present. In terms of their mass m_{ν_s} , find the present value of their density parameter $\Omega_{\nu_s,0}$. (You may adopt $H_0 = 70 \text{ km/s/Mpc}$ and $T_{\gamma 0} = 2.725 \text{ K}$). If observations require that the upper bound to their present density is $\Omega_{\nu_s,0} \leq 0.25$, find the upper bound to the ν_s mass. [10]

Problem 3 :

Suppose there is a massive, spin 1/2 Fermion X , which was in equilibrium during the very early evolution of the Universe. Suppose that $m_X = 2$ GeV and that the annihilation rate factor $\beta \equiv \langle \sigma_{ann} v \rangle = 1.0 \times 10^{-15} \text{ cm}^3/\text{s}$.

a) Find the temperature T_* at which the abundance of the X particles begins to depart from their equilibrium abundance ($n_{X*} \gtrsim n_{Xeq*}$). [10]

HINT: To find T_* , you will need to find x_* , which also involves knowing (guessing) a value for g_* . For example, guess a range for T_* , find g_* in this range and solve for x_* . With this value of x_* , find T_* and g_* and compare them to your original guesses. If necessary, iterate by repeating these steps until x_* , T_* , and g_* are internally consistent.

b) If the present ratio of **nucleons** to photons (CBR) in the Universe is $(n_N/n_\gamma)_0 = 6 \times 10^{-10}$, compare (find the ratio of) the number density of the X s to that of the nucleons in the present Universe. [15]

Problem 4 : WIMPS

Suppose the Dark Matter in the Universe is due to a neutral, weakly interacting massive particle (WIMP), a spin 1/2 Fermion X . Suppose that the annihilation rate factor for the X is $\beta = 5 \times 10^{-24} m_X^{-2} \text{ cm}^3/\text{s}$, where m_X is in GeV. (You may assume that $H_0 = 70 \text{ km/s/Mpc}$ and $T_{\gamma 0} = 2.725 \text{ K}$).

a) If $\Omega_{X0} = 0.20$, find x_* , T_* , and m_X . [15]

HINT: To find x_* , T_* , and m_X you will need to iterate, similar to Problem 3. For example, use the relation for Ω_{X0} to find a relation among x_* , g_* , and m_X . This will enable you to eliminate m_X in the expression for x_* . Then guess a value for g_* and solve for x_* and m_X and find T_* . See if the corresponding value of g_* is consistent with your guess. If necessary, iterate by repeating these steps until x_* , T_* , and g_* are internally consistent.

b) The non-relativistic X s (and \bar{X} s) will cluster in Dark Matter halos at present, with local densities much higher than their mean density in the Universe ($n_X \gg n_{X0}$). What enhancement factor, n_X/n_{X0} , would be needed in these Dark Matter halos if the annihilation rate (per X) were to be faster than the present expansion rate of the Universe (H_0)? [10]