Physics 880A20 COSMOLOGY Problem Set 6 October 30, 2006

Note: The numbers in [ ] are the point totals for each part of each problem.

Problem 1 :

Suppose the three "standard" neutrinos (ν_e, ν_μ, ν_τ) were Bosons instead of Fermions. For temperatures in the range $m_e \ll T_\gamma \ll m_\mu$, find:

a) The new effective number of degrees of freedom g'_{eff}. [5]

b) The expansion rate "speed-up" factor, $S \equiv H'/H$ (compared to the standard case where neutrinos are fermions). [5]

c) Assume that the bosonic neutrinos decouple prior to $e^{\pm}$ annihilation (as do the standard fermionic neutrinos). Find the ratio at present of neutrino temperature to photon temperature. [5]

d) Find the expansion rate "speed-up" factor, $S \equiv H'/H$, during the post-$e^{\pm}$ annihilation, radiation dominated era. [5]

Problem 2 : Sterile Neutrino

Suppose there is a fourth flavor of massive, but light ($m_{\nu_s} \ll m_e$), neutrinos, ν_s, which through mixing (oscillations) with the standard neutrinos are in equilibrium at temperatures $T_\gamma \gtrsim T_{\nu dec} \gtrsim m_e$. Assume that this 4th ("sterile") neutrino decouples along with the 3 standard neutrinos at $T_\nu \sim 2-3$ MeV.

a) For $m_e \ll T_\gamma \ll m_\mu$, find g'_{eff} and the speed-up factor S. [10]

b) Find the ratio **by number** of the ν_s to photons after $e^{\pm}$ annihilation. [5]

c) Suppose the ν_s have a mass, m_{ν_s}, and they are non-relativistic today. If the present value of the ν_s mass density parameter is $\Omega_{\nu_s 0} \leq 0.24$, find the upper limit to m_{ν_s} (you may assume that $H_0 = 72$ km/s/Mpc and that $T_{\gamma 0} = 2.725$ K). [15]

<u>Problem 3</u> :

Suppose there is a very weakly interacting scalar particle X which is massive but light ($m_X \ll 1$ MeV). Suppose that these X particles decouple at a temperature of $T_{X\,dec} \approx 500$ MeV (a temperature above that of the quark-hadron transition).

a) List the particles which are present and extremely relativistic at $T_{X\,dec}$ and evaluate $g_I(T_{X\,dec})$. [10]

b) For $m_e \lesssim T_\gamma \lesssim T_{\nu dec}$ find ΔN_ν, the "equivalent number of extra neutrinos" contributed by the X particles. [10]

c) Suppose that at present the X particles are non-relativistic. If the present value of the mass density parameter due to the X particles is $\Omega_{X0} \leq 0.24$, find the upper bound on the X mass (you may adopt $H_0 = 72$ km/s/Mpc and $T_{\gamma,0} = 2.725$ K). [10]