Physics 880A20 COSMOLOGY Problem Set 5 October 23, 2006

Note: The numbers in [ ] are the point totals for each part of each problem.

Problem 1 :

Consider the standard ΛCDM cosmological model with $\Omega_{\rm M} = 0.24$ and $\Omega_\Lambda = 0.76$; you may adopt $H_0 = 72$ kms^{-1}Mpc^{-1}. Assume that $\Omega_{\rm R} = 8.0 \times 10^{-5}$ for the current density in relativistic particles ("radiation").

a) Find the redshift of equal matter and radiation densities, z_{eq}. [5]

b) Find the age of the Universe, t_{eq}, at z_{eq}. [5]

c) "Recombination" occurs for $z_{rec} = 1100$. Find the age of the Universe, t_{rec}, at z_{rec}. [5]

d) Find the proper radial distance to the horizon at recombination, $R_{\rm H}(t_{rec})$, in units of the present Hubble distance c/H_0. [10]

e) Identify each of the horizon-sized volumes from part d) with a comoving volume. How many of these comoving volumes are today (t_0, $z = 0$) within one "Hubble volume" ($\equiv \frac{4\pi}{3}(\frac{c}{H_0})^3$)? [5]

Problem 2 :

For the Steady State cosmological model it is **assumed** that the number **densities** of particles, photons included, remains constant as the Universe expands (and, in particular, the photons redshift). For the Steady State model suppose that at some time t_e in the past, at redshift z, a **differential** spectrum of radiation $n(\nu_e, t_e)$ (number of photons per unit frequency interval, per unit volume) is emitted.

a) For a general spectrum, $n(\nu_e, t_e)$, relate the brightness temperature at time t_e ($T_b(\nu_e, t_e)$) to $n(\nu_e, t_e)$ and also to the brightness temperature this spectrum evolves to at present, ($z = 0$, t_0), $T_b(\nu_0, t_0)$. [10]

b) Suppose that the spectrum created at time t_e is Black Body at temperature T_e. Find $T_b(\nu_0, t_0)$. Is the spectrum Black Body at t_0? [10]

c) In the Rayleigh-Jeans limit, $h\nu_0 \ll kT_0$, where $T_0 \equiv T_e/(1+z)$, find the ratio of $T_b(\nu_0, t_0)$ to T_e. [5]

d) In the high frequency limit $h\nu_0 \gg kT_0$, where $T_0 \equiv T_e/(1+z)$, find the ratio of $T_b(\nu_0, t_0)$ to T_e. [5]

Problem 3 :

During the early Radiation Dominated evolution of the Universe there was a time when the temperature (thermal energy) was in the range $m_\mu < T_\gamma < m_\pi$ ($105 < T_\gamma(MeV) < 135$). Assuming the known particles (including three flavors of neutrinos) are present, find

a) The age-temperature relation (with t in sec and T in MeV). [5]

b) The expansion rate-temperature relation (with H in $\sec^{-1}$ and T in MeV). [5]

c) The energy density, $\rho_{\rm R}$, (in units of GeV/fm^{-3}) as a function of the temperature (in units of MeV). [5]