Physics 880A20 COSMOLOGY Problem Set 4 October 16, 2006

Note: The numbers in [ ] are the point totals for each part of each problem.

Problem 1 :

Assume that the Universe is flat ($k = 0$) and that the energy density is dominated by a field (X) whose equation of state is: $w_X = -1/3$.

a) For this model find ρ_X as a function of the scale factor (or, of $x \equiv a/a_0$) and also as a function of the redshift. [5]

b) For this model find x versus t in terms of H_0 and/or t_0. [5]

c) For this model find the present age (t_0) in terms of the Hubble age H_0^{-1}. Also, find the deceleration parameter q for this model. [5]

d) For this model find the lookback time ($\Delta t \equiv t_0 - t_e$), in units of the Hubble time, versus redshift. [5]

e) For this model find the distance modulus ($m - M$) as a function of redshift and evaluate it for $z = 0.04$ and for $z = 0.4$. You may assume that $H_0 = 70$ km/s/Mpc. [10]

Problem 2 : Dark Energy

Assume that the Universe (near the present epoch) consists of two components, "ordinary" (non-relativistic) matter ($w_{\rm M} = 0$) and "dark energy" (X) with an equation of state $w_X = -1$. For each of these components assume that the present values of the density parameters are $\Omega_{\rm M0} = 1/3$ and $\Omega_{X0} = 2/3$.

a) Find the redshift, z_{eq}, at which $\rho_{\rm M} = \rho_X$. [5]

b) Find the redshift, z_*, at which the acceleration of the expansion of the Universe vanishes. [5]

c) For this model find the present value of the deceleration parameter q_0. [5]

d) For this model consider the future evolution ($t >> t_0$, $a >> a_0$) and find the proper radial distance to the event horizon $R_{\rm E}(t)$. [10]

Problem 3 : Phantom Energy (Big Rip)

Assume that the Universe (near the present epoch) consists of two components, "ordinary" (non-relativistic) matter ($w_{\rm M} = 0$) and "phantom energy" (X) with an equation of state $w_X = -4/3$. Assume that for each of these components the present values of the density parameters are $\Omega_{\rm M0} = 1/4$ and $\Omega_{X0} = 3/4$.

a) Find the redshift, z_{eq}, at which $\rho_{\rm M} = \rho_X$. [5]

b) Find the redshift, z_*, at which the acceleration of the Universe vanishes. [5]

c) For this model find the present value of the deceleration parameter q_0. [5]

d) This open, ever expanding model has a finite lifetime ($a \to \infty$ for $t = t_\infty < \infty$). Starting at a time in the future, $t_1 >> t_0$, how much time does it take ($\Delta t \equiv t_\infty - t_1$) for the "big rip" ($a \to \infty$) to occur? [10]

e) Consider the future evolution of this model ($t_1 >> t_0$, $a_1 >> a_0$) and find the proper radial distance to the event horizon $R_{\rm E}(t_1)$. Find the limit of $R_{\rm E}(t_1)$ as $a_1 \to \infty$. [10]