Physics 880A20 COSMOLOGY Problem Set 2 October 2, 2006

Note: The numbers in [ ] are the point totals for each part of each problem.

Problem 1 :

This is a problem in **Newtonian** mechanics. Consider a uniform (i.e., homogeneous) sphere of mass M and initial radius R_0. The system is released from rest at $t = 0$ and collapses under its own gravity.

a) In terms of M and R_0, find the time, t_{coll}, for the sphere to collapse to a point, $R = 0$. [10]

b) Suppose the **same** uniform sphere of mass M were released from $R_0' = 2R_0$. Find the ratio of the new collapse time t_{coll}' to t_{coll} (from part a). [5]

c) In the limit $R \ll R_0$, where $R = R(t)$ is the radius of the sphere at $t > t_0$, find the deceleration parameter q. (Recall that $q \equiv \frac{-1}{H^2}[\frac{1}{R}(\frac{d^2R}{dt^2})]$.) [5]

Problem 2 :

There is an interesting class of cosmological models for which the 3-space curvature may be neglected ($k = 0$) and the time-evolution of the scale factor is a power law ($a \propto t^\alpha$, where $0 < \alpha < 1$). For this class of models find (in terms of α):

a) The present age (t_0) and the look-back time (Δt), in terms of the "Hubble age" H_0^{-1}, as a function of the redshift z. [5]

b) The deceleration parameter as a function of α ($q = q(\alpha)$). [5]

c) Is there an event horizon for this class of models? (Justify your answer.) [5]

d) The comoving radial coordinate of the (particle) horizon as a function of time ($\Theta_\mathrm{H}(t)$) and as a function of the redshift ($\Theta_\mathrm{H}(z)$) (where $z = z(t)$). [5]

e) The proper radial distance to the horizon (R_H) as a function of time. [5]

f) The luminosity distance as a function of the redshift ($\mathrm{d_L}(z)$). Expand this result to order z^2 and express the result in terms of the deceleration parameter (instead of α). [10]

Problem 3 :

For the same, spatially flat, power-law cosmological model as in Problem 2, consider a comoving volume out to $\Theta_\star$ corresponding to a redshift $z_\star$, when the age of the universe was $t_\star$ (i.e., the photons emitted from $\Theta_\star$ at $t_\star$ are received at $\Theta = 0$ at the present time, t_0). For conserved particles (e.g., CMB photons) find:

a) The total number of particles in the comoving volume at present (t_0) out to $\Theta_\star$, $\mathrm{N}_\star(t_0)$. [5]

b) The total number of particles which were inside the horizon when the light was emitted at time $t_\star$, ($\mathrm{N}_\mathrm{H}(t_\star)$). [5]

c) Compare $\mathrm{N}_\star(t_0)$ and $\mathrm{N}_\mathrm{H}(t_\star)$ (i.e., find the ratio $N_\star(t_0)/N_\mathrm{H}(t_\star)$ and determine if it is < 1 or > 1). [10]