Physics 880A20 COSMOLOGY Final Exam Problem Set November 27, 2006

- **For this Final Exam Problem Set you should work on your own – no group work please. You may (should!) consult the class notes and any textbooks, but if you should find the solution to a problem elsewhere, you still must show your work.**

- **The Final Exam Problem Set is due in my office by 3:30 on Monday, December 4.**

- **There are only 2 problems for a total of 75 points.**

- **Unless otherwise indicated, you may take $H_0 = 70$ km/s/Mpc.**

Problem 1 : **50 points**

Consider an exotic form of matter, "X-matter", whose equation of state is $w_X = +1$.

a) In terms of the present X-matter density, ρ_{X0}, find the X-matter density as a function of redshift, $\rho_X = \rho_X(z)$. [5]

b) Consider a **flat** ($k = 0$) universe with "cold" (NR) matter (M) and X-matter (no Dark Energy). You may assume that radiation is sub-dominant today, with a present density parameter value of $\Omega_{\rm R} = 8.5 \times 10^{-5}$. In terms of the present X-matter density, ρ_{X0}, find the deceleration parameter as a function of redshift, $q = q(z)$. Evaluate q_0 (in terms of the present X-matter density parameter Ω_X). [10]

c) In terms of the present X-matter density parameter Ω_X (and H_0), find the age – redshift relation for this cosmological model. Find the present age, t_0, if $\Omega_X = \Omega_{\rm M}$. [10]

d) Was this universe (with $\Omega_X = \Omega_{\rm M}$) ever radiation-dominated in the past? Will it be radiation-dominated in the future? If at present, $T_{\gamma 0} = 2.725$ K, how old is this universe when BBN (normally) ends at $T_\gamma = 30$ keV? [10]

e) In this part assume that $\Omega_{\rm M} = 0$ and find the luminosity distance – redshift relation, $d_{\rm L} = d_{\rm L}(z)$. Evaluate the distance modulus, $m - M$, for $z = 1$. [15]

Problem 2 : **25 points**

Consider a very massive fermion X (you may assume $g_X = 2$) whose thermally averaged annihilation rate factor depends on its mass and on the temperature as: $\beta = 2 \times 10^{-22}(m_X^3 T)^{-1/2}$ cm^3/s. m_X and T are in GeV.

a) If $\Omega_{X0} = 0.25$, find m_X.   [20]

b) If the present baryon (nucleon) density parameter is $\Omega_{\mathrm{B0}} = 0.045$ and $\Omega_{X0} = 0.25$, find the ratio, by number, of X particles ($X + \bar{X}$) to nucleons. You may assume the nucleon/baryon mass is $m_{\mathrm{N}} = 0.94$ GeV. (Note: Even if you were unable to find m_X in part a), you could answer this part in terms of the unknown (or wrong!) value of m_X.)   [5]

[You will probably solve for m_X iteratively. I do **NOT** need to see all your iterations. Just show me the equations you use to solve for x_* and for Ω_{X0}, in terms of m_X and g_*, and then show me your answers for x_*, T_*, g_*, and m_X. Check that they are consistent!]

HAPPY HOLIDAYS!