PHYSICS 631 : SET # 9 Due 2nd November, 2009

1.(3pts.)

(a) Griffiths Prob. 2.26

Just plug $f(x) = \delta(x)$ into Griffiths eq.[2.103]. You should, however, read the comments on page 77 after eq.[2.144].

(b) What is the Fourier Integral Representation of $\delta(x - x_0)$?

(c) Suppose that at time $t = 0$, a position measurement is made on a particle and $x = x_0$ is found. Assume that measurement was precise enough and the wave function immediatly following it is well approximated by a δ-function (or a very very narrow Gaussian). Argue, that there is equal probability to find any value of the momentum immediatly afterwards. This is, of course, in agreement with the Uncertainty Principle.

2.(2pts.)

The probability density $\rho(x,t) \equiv \Psi^*\Psi$ and the probability current density $J(x,t) \equiv \frac{\hbar}{2mi}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \frac{\partial\Psi^*}{\partial x}\Psi\right)$ obey the continuity equation

$$\frac{\partial\rho}{\partial t} + \frac{\partial J}{\partial x} = 0$$

What is the probability current density, J, for ,

$$\Psi(x,t) = \left[Ae^{ikx} + Be^{-ikx}\right] e^{-i\hbar k^2 t/2m}$$

Verify that the continuity equation is trivially satisfied.

2.(5pts.)

Consider our familiar "particle in a box", $0 \le x \le a$, system again. At $t = 0$ the wave function is given by $\Psi(x,0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$, with ψ_i = energy eigenfunctions. Calculate the probability density and the probability current density for the system for general $t > 0$. Verify that the continuity equation is satisfied.

It might simplify expressions to write $E_1/\hbar$ and $E_2/\hbar$ in terms of $\omega \equiv \frac{\pi^2\hbar}{2ma^2}$.

PHYSICS 631 : SET # 10 Due 5th November, 2009

2.(10pts.)

In class we considered the standard step potential of Fig.(a) and found the transmission coefficient $T = \frac{4k_1k_2}{(k_1+k_2)^2}$ with $k_1^2 = \frac{2m}{\hbar^2}E$ and $k_2^2 = \frac{2m}{\hbar^2}(E - V_0)$.

Consider the "step-down" potential of (b) again for $E > V_0$. We will assume as usual that the incident particle comes in from the left (from $x = -\infty$).

(i) Find the solutions ϕ_I and ϕ_{II} to the energy eigenvalue equation in regions I and II for potential (b). Carry out appropriate matchings and show that the transmission coefficients, T, are identical for cases (a) and (b).

(ii) What are the probability current densities J_I and J_{II} in regions I and II of (b)?

(iii) Given the "continuity equation", how should J_I and J_{II} be related to each other? Verify that your answer to (ii) satisfies this relation.

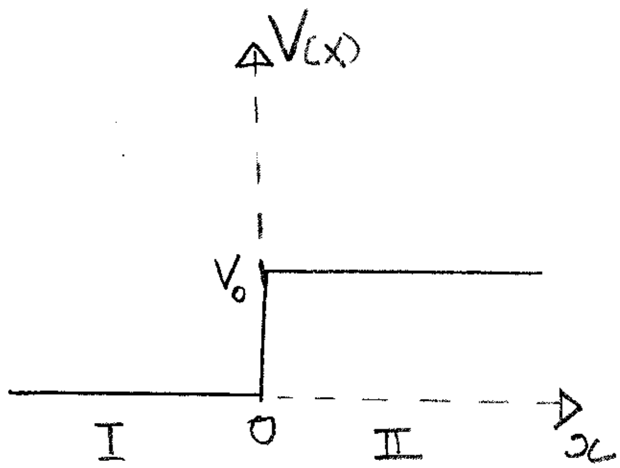

Fig. a

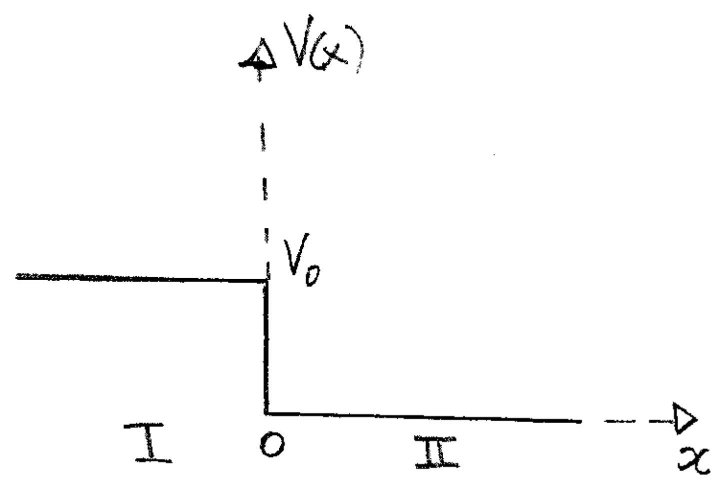

Fig. b