PHYSICS 631 : SET # 5 Due 12th October, 2009

1.(5pts.)
Consider a particle in a box centered around the origin, i.e.
$V(x) = 0$ for $-a/2 \leq x \leq a/2$ and ∞ otherwise.

Solve separately for the <u>even</u> ($\psi_{even}(-x) = \psi_{even}(x)$) and <u>odd</u> ($\psi_{odd}(-x) = -\psi(x)$) solutions to the energy eigenvalue equation $\hat{H}\psi = E\psi$ for this system.
Verify that you get the same set of eigenvalues $\{E_n\}$ as for the $0 \leq x \leq a$ box.
Show that the even functions $\psi_{n=2l-1}(x) = \psi_{2l-1}(-x)$ and odd functions $\psi_{n=2l}(x) = -\psi_{2l}(-x)$ are given by,

$$\psi_{2l-1} = \sqrt{\frac{2}{a}}\, cos\left(\frac{(2l-1)\pi}{a}x\right), \qquad \psi_{2l} = \sqrt{\frac{2}{a}}\, sin\left(\frac{2l\pi}{a}x\right).$$

$l = 1, 2, 3, ...$

Plot the 4 eigenfunctions corresponding to the four lowest energy levels and verify that they agree with those for the $0 \leq x \leq a$ box (up to overall signs).

2.(5pts.)
(a) Griffiths Prob. 2.12

(b) Check your answer for $< \hat{x}^2 >$ by doing an explicit integral for the case $n = 1$, i.e. <u>without</u> taking advantage of raising and lowering operators.
$\phi_1(x) = (\frac{m\omega}{\pi\hbar})^{1/4}\frac{1}{\sqrt{2}}(2\xi)e^{-\xi^2/2}$ with ($\xi = \sqrt{\frac{m\omega}{\hbar}}x$).

PHYSICS 631 : SET # 6 Due 15th October, 2009

1.(2pts.)
The operator $\hat{x}$ manifests itself in different ways. For instance,

$$\hat{x}\phi_1 = x\,\phi_1 \qquad (5)$$

or

$$\hat{x}\phi_1 = \sqrt{\frac{\hbar}{2m\omega}}\,[\hat{a} + \hat{a}^\dagger]\,\phi_1 = \sqrt{\frac{\hbar}{2m\omega}}\,[\phi_0 + \sqrt{2}\phi_2] \qquad (6)$$

where use was made of $\hat{a}\phi_n = \sqrt{n}\phi_{n-1}$ and $\hat{a}^\dagger\phi_n = \sqrt{n+1}\phi_{n+1}$. Verify that the RHS of (5) and (6) are consistent by explicitly plugging in the functions $\phi_1(x)$, $\phi_0(x)$ and $\phi_2(x)$. See Griffiths eq.[2.85] for energy eigenfunctions.

2.(2pts.)
A particle in a harmonic oscillator potential has initial wave function ,

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}}(\phi_7 + \phi_8)$$

with ϕ_n = eigenfunctions of the harmonic oscillator Hamiltonian.

What are $\Psi(x,t)$ and $(\hat{a}\,\Psi(x,t))$ at $t > 0$?
$\hat{a}$ is the usual lowering operator and you are asked for the result of applying it to $\Psi(x,t)$.
What is $\int \Psi^*(x,t)\,\hat{a}\,\Psi(x,t)\,dx$?
Do you think that $\hat{a}$ can correspond to a physical observable ?
(can it be measured ?)

3.(6pts.)
Griffiths Prob. 2.13
In part (c), rewrite $\hat{x}$ in terms of lowering and raising operators.