PHYSICS 631 : SET # 1 Due 28th September, 2009

1.(3pts.)
Read and work through Griffiths §1.3 and then do Griffiths Prob. 1.1

2.(3pts)
This exercise reviews important properties of complex numbers that will be needed this Quarter.
(a) Listed are several complex numbers "z".

$z = a + ib$, $z = \rho\, e^{i\phi}$, $z = \frac{e^{i\phi}}{\sqrt{a+ib}}$, $z = \frac{a+ib}{c+id}$, $z = ae^{i\alpha} + be^{i\beta}$ and $z = c_1 e^{i\alpha} + c_2 e^{i\beta}$

(a,b, c, d, ρ, ϕ, α, β are all real numbers and c_1, c_2 are complex)

For each one, give its

complex conjugate z^*

absolute value squared $|z|^2$

(b) Recall the important formula : $e^{i\phi} = cos(\phi) + i\, sin(\phi)$

Verify,

$$\begin{aligned} e^{i\pi} &= -1 \\ e^{i\frac{\pi}{2}} &= i \\ \sqrt{i} &= \frac{1}{\sqrt{2}}(1+i) \end{aligned}$$

What is $e^{-i\phi}$? (in terms of cos and sin's ?)
What are $cos(\phi)$ and $sin(\phi)$ in terms of exponentials ?

3.(4pts.) Griffiths Prob. 1.4

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PHYSICS 631 : SET # 2 Due 1st October, 2009

1.(4pts.) The commutator of two operators $\hat{A}$ and $\hat{B}$ is defined as : $[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}$.

(a) Consider three operators, $\hat{A}$, $\hat{B}$ and $\hat{C}$. Verify

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$$

and

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

(b) What is $[\hat{x}, \hat{H}]$ where $\hat{H} = \hat{p}^2/2m + V(\hat{x})$? Do the calculation in two ways :

(i) put in explicitly $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ etc. and let
$[\hat{x}, \hat{H}]$ act on an arbitrary wave function $\psi(x)$

and

(ii) exploit the formulas in part (a) which will save you the trouble of having
to introduce a trial $\psi(x)$.

2.(4pts.)

(a) Let $\hat{A}$ = operator that is not an explicit function of time, i.e. $\frac{\partial \hat{A}}{\partial t} = 0$.
Show that

$$\frac{d}{dt} < \hat{A} >= \frac{1}{i\hbar} < [\hat{A}, \hat{H}] > \qquad (1)$$

Start from the LHS with $\langle \hat{A} \rangle \equiv \int_{\infty}^{\infty} \Psi^* \hat{A} \Psi dx$ and use the Schrödinger equation. At some point you will have to integrate by parts twice, throwing away surface terms (since Ψ vanishes at $x \to \pm\infty$).

(b) Let $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$. Use the above eq.(1) to derive "Ehrenfest's Theorem",

$$\frac{d}{dt}\langle \hat{x} \rangle = \frac{1}{m}\langle \hat{p} \rangle \qquad (2)$$

$$\frac{d}{dt}\langle \hat{p} \rangle = -\langle \frac{\partial \hat{V}}{\partial x} \rangle \qquad (3)$$

Note: eq.(2) was derived already in class without the help of (1).

3.(2pts.)

Verify that a Gaussian wave function,

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha(x-a)^2/2}$$

leads to the smallest joint uncertainty $\sigma_x \times \sigma_p$ allowed by the Uncertainty Principle.
i.e. calculate $\langle \hat{x} \rangle$, $\langle \hat{p} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p}^2 \rangle$, σ_x, σ_p and the product $\sigma_x \sigma_p$ for this system.
It might be useful to shift integration variable from x to $y = x - a$.