The Effect of Neutrino Degeneracy on Big Bang Nucleosynthesis

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Abstract

The standard model of the Big Bang makes definite predictions about primordial abundances of light elements, up to ^{7}Li. The only free parameter in standard BBN is the baryon to photon ratio, η, which is rather tightly constrained. This paper presents a review of the standard model and proposes that by allowing for ν_e degeneracy the range of acceptable values for η can be expanded. All modeling of element abundance is done using the Wagoner code, the most widely used code for such predictions.

I The Standard Model [1,2,3]

About three minutes after the Universe began its initial expansion; it had cooled to the point where nucleosynthesis could begin in earnest. To understand how this occurred, one must look a little further back in time to about 10^{-2} seconds after the Big Bang.

At 10^{-2} seconds, The universe consisted of a soup of relativistic particles composed primarily of protons, neutrons, electrons, positrons, neutrinos and antineutrinos. At this temperature, T ~ 10 MeV all the particles were in thermal equilibrium. That is to say that the particles interacted frequently enough to transmit any sensible difference across the volume of the Universe at that time. The interactions to which we refer are the weak interactions, an example of which is $\nu + n \leftrightarrow p + e^-$. This equilibrium, as it refers to the baryons, fixes the neutron to proton ratio (n/p hereafter.) The relation is dictated by $n/p = e^{-Q/T}$ [1] where Q is the mass difference between the neutron and the proton in MeV and T is the temperature, also in MeV. At 10 MeV this ratio is about 1.

The condition of equilibrium demands a rapid reaction rate. This rate is set by, among other things the cross section of the particles in question and number densities of the particles. Both of these factors depend on temperature, cross section (σ) ~ T^2 and number density (n) ~ T^3 with the result that the reaction rate varies as T^5. As long as this rate is faster than the expansion rate, our criterion of rapid reactions is maintained. [3]

The expansion rate is given by the Friedmann equation:

$$\left(\frac{\dot{R}}{R(o)}\right)^2 + \frac{k}{R(o)^2} = \frac{8pGr(o)}{3}\left(\frac{R(o)}{R(t)}\right)^2$$

Where: $\left(\frac{\dot{R}}{R(o)}\right)$ = is the Hubble parameter (H), and represents the expansion rate of the Universe, and ρ represents the energy density of the universe. These are the only two terms that concern this discussion. We can see that the expansion rate is proportional to the energy density. This density is in turn proportional to T^4. Therefore, $H \sim T^2$.

It is easy to see that lowering the temperature has a much greater effect on the reaction rates than it does on the expansion rate, and if the reactions are initially faster than the expansion, then there will be some temperature T_{eq} where the reaction rate is equal to the expansion rate. This event is known as freezeout, and when this temperature threshold is crossed, the equilibrium value of n/p no longer maintains, but is fixed. Current calculations put that ratio at 1/6 and the temperature at 0.8 MeV. Figure 1 shows a plot of the equilibrium value of n/p.

This temperature is still too high for nucleosynthesis to occur. The universe further cools to about 0.3 MeV, and the process of building up nuclei begins. Because of the delay, free neutron decay has lowered n/p to about 1/7, and this ratio is what determines primordial abundances.

It should be noted that the only free parameter in the standard model is the ratio of the number densities of baryons to photons, η. All predicted abundances can be expressed as functions of η, and the observed abundances make a useful probe of this value. For the purposes of this paper, the element abundances used were as follows:[7]

$$.228 < {}^4He < .248 \text{ by mass fraction}$$

$$2\text{x}10^{-5} < {}^2H < 5\text{x}10^{-5} \text{ by ratio of number densities}$$

$$1\text{x}10^{-10} < {}^7Li < 4\text{x}10^{-10} \text{ by ratio of number densities.}$$

Both number density ratios are with respect to H, i.e. n(^{7}Li)/n(H) ~ 10^{-10}

These constraints are based upon a myriad of observational evidence and techniques, and are not as restrictive as some used by other groups. As we will see, they place fairly tight constraints on η and one of the purposes of this work is to open up the range of η.

Figure 2 shows a plot of primordial element abundances vs. η.

II Neutrino Degeneracy

What do we mean when we talk of neutrino degeneracy? Unlike the quantum mechanical definition of degeneracy (multiple quantum states with the same energy), we are referring to an asymmetry between the numbers of neutrinos and antineutrinos. Given that most of the Universe is composed of matter (we think…) this is not an unreasonable condition.

Neutrino degeneracy affects the outcome of nucleosynthesis in two primary ways.[9] First, the degeneracy alters the equilibrium value of n/p for a given temperature. This new equilibrium value is given by $n/p = e^{-(Q-f)/T}$ [1] where ϕ is the degeneracy parameter. It is the essentially the chemical potential divided by $k_B T$.

The other effect is to speed up the expansion rate of the Universe. Recall that the square of the expansion rate depends on energy density. Degenerate neutrinos increase the energy density and therefore the expansion rate. This results in the expansion rate overtaking the reaction rates at an earlier time. Earlier freezeout again alters n/p.

III <u>The Method</u>

The calculations that predict nucleosynthesis abundances are arduous at best. The Wagoner [8] code, written in1973 and updated periodically is the standard means of calculating these results. In its standard form, it cycles through successive values of η and calculates ^{4}He, D, ^{3}He, and ^{7}Li. We modified the code to cycle through successive values of ϕ from –10 to 10 in steps of 0.5 for a specified value of η. This produced 41 values of each abundance, discarding helium-3.

The thrust of this research is to show that with some arbitrary spatial distribution of ϕ we can still achieve observed abundances of elements. It is at this point that we turned to a linear programming method. Since there are an infinite number of possible functions to choose from, finding a function that satisfies our constraints would be time consuming at best. The approach used here is similar to that of Leonnard and Scherrer [] to explore the possibility of an inhomogeneous η.

In general, linear programming can be used to maximize or minimize a linear function subject to a number of constraints. In order for a maximum or minimum to exist, a solution vector for the constraint equalities and inequalities must exist, i.e. the constraints must enclose some nonzero region of the variable space. In order to use linear programming, we must couch the problem in discrete terms. Borrowing the results from [5] we find that our arbitrary function will be a linear combination of δ functions. This by no means represents the actual distribution of ϕ, but its existence shows that there is some distribution that satisfies the constraints and maximizes the function.

The method used is known as the simplex method, and the FORTRAN [4] code used to implement it was taken from Press et. al. The simplex method demands that the constraints be recast as equalities. Our abundance constraints then translate into 6 constraint equations. A seventh constraint is added when we require that our variables add up to one. Each equation has 41 variables in it, one for each bin corresponding to the degeneracy. For example, the first constraint equation would be :

$$.228 = a_{(-10)}Y_{(-10)} + a_{(-9.5)}Y_{(-9.5)} + \ldots + a_{(10)}Y_{(10)}$$

where $a_{(j)}$ is the j^{th} variable and $Y_{(j)}$ is the j^{th} helium abundance. The subscript refers to the degeneracy bin. In the language of linear programming, the abundance produced by the Wagoner code are the coefficients and the a values are the variables to be adjusted to maximize the solution.

It turns out that we don’t really care what the maximum value of our function is, but rather that our 7 by 41 matrix is consistent. Fortunately, the simplex code used contains a flag that states whether or not a solution even exists. An arbitrary function was created and the code was run, first Wagoner for a specific η, and then the simplex to see if the coefficients formed a consistent system of equations. This process was repeated for successively lower η until the system was inconsistent. The lowest value of η that yielded a consistent set was taken to be the lower bound. The process was repeated for the upper bound The Wagoner code was also ran for a no degeneracy case to establish standard model bounds on η. These were found to be in excellent agreement with current literature such as Kolb and Turner.[1]

IV Results

After running the codes numerous times, we arrived at four ranges for η. These ranges correspond to the following conditions: no degeneracy, inhomogeneous degeneracy, and two cases of homogeneous degeneracy. The table below summarizes these results.

Degeneracy Type	Lower Bound of η	Upper Bound of η
None	3.81×10^{-10}	5.02×10^{-10}
Inhomogeneous	3.22×10^{-10}	8.60×10^{-10}
Homogeneous (<0)	3.5×10^{-10}	4.96×10^{-10}
Homogeneous (>0)	4.66×10^{-10}	5.0×10^{-10}

One possible problem with the standard model is that by some calculations it overproduces ^{4}He by >10%.[6] By expanding the range of η, a possible resolution to this "crisis" may be found.

A few words about errors are in order. Much of the error in the predictions arises from two specific areas, the free neutron half-life and the reaction rates for all of the nuclear interactions. For helium, the errors are relatively small, but as the nuclides get larger, the uncertainty grows. For lithium, the predicted uncertainty could be as large as 50%. This large number arises not only from the factors mentioned above, but also from the fact that there are two distinct lithium formation mechanisms, one predominant at lower values of η and the other at higher values. These two mechanisms are responsible for the "lithium trough" in figure 2. Uncertainty and errors in observations have been incorporated into the ranges of accepted values for each specie.

V **Figures**

1) Equilibrium value of n/p vs. temperature, 0 degeneracy case

2) Predictions of the standard model [1]

Figure 1

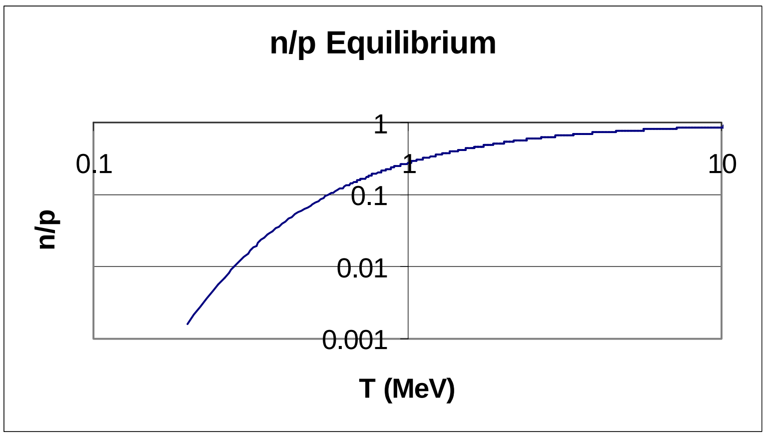
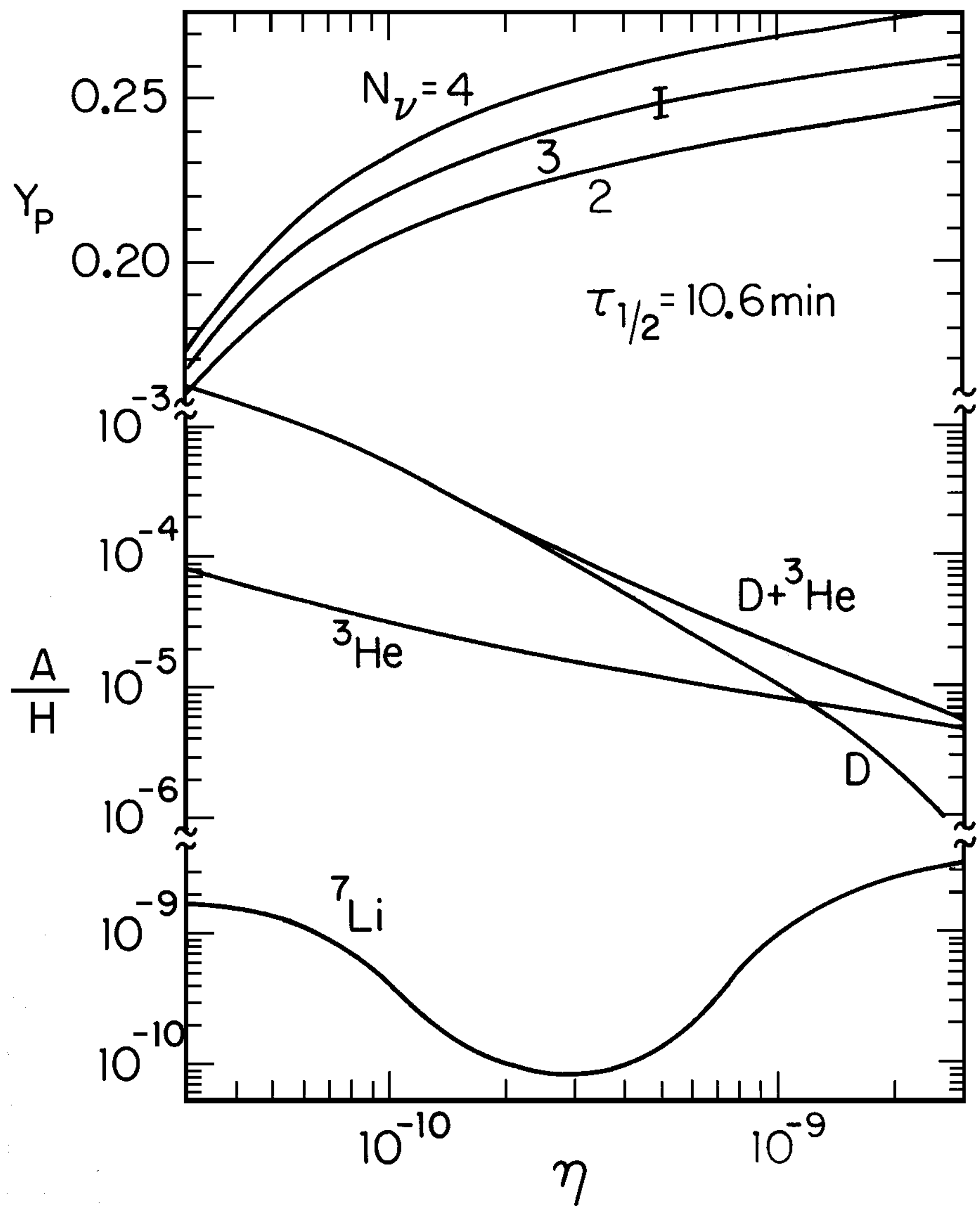


Figure 2



0.25
0.20
Y_P
N_ν = 4
3
2
I
τ_1/2 = 10.6 min
10^-3
10^-4
10^-5
10^-6
A/H
D+^3He
^3He
D
^7Li
10^-9
10^-10
10^-10
10^-9
η

VI References

[1] E. Kolb, M. Turner, *The Early Universe*, Addison-Wesley, 1990

[2] S. Weinberg, *Gravitation and Cosmology*, Wiley, 1972

[3] J. Bernstein, *An Introduction to Cosmology*, Prentice Hall,1995

[4] Press, et. al., *Numerical Recipes*

[5] R. Leonard, R. Scherrer, Astrophys. J. **463** 420-423

[6] N. Hata, R. Scherrer et. al., hep-ph/950319/v2, 7 Nov. 1995

[7] Discussion with R. Scherrer

[8] The version of the code used was obtained from R. Scherrer

[9] H. Kang, G.Steigman, Effects of Neutrino Degeneracy on Nucleosynthesis.

VII Acknowledgements

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