Development of a Method for Measuring the $M^2$ Parameter of a Laser

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Abstract: The $M^2$ factor has become a popular and important parameter, used to characterize the quality of a laser beam. A procedure for making an accurate $M^2$ measurement of a laser is developed and described. Also discussed are several methods of measurement which did not yield adequate results.

Introduction

In most applications of lasers, control or at least knowledge of the beam’s spot size and intensity is crucial to achieving the desired result. However, the diffraction limited spot size of any real beam will be somewhat larger, and thus the intensity lower, than theory based on a perfect Gaussian beam would predict. Having a parameter that indicates quantitatively how a laser deviates from the ideal is more useful than a single determination of spot size, since it can be then applied to any optical setup involving the beam without having to take new measurements. In this particular case, the laser in the laboratory is used in multi-photon ionization experiments, employing time-of-flight measurements to determine the electron spectrum of the sample. The beam’s spatial intensity distribution in the interaction region is modeled as a Gaussian pulse in the algorithms used to interpret the data. Thus, the motivation to develop beam characterization techniques comes from a desire to know the extent to which the pulse can be treated as a Gaussian and to understand the limits that beam quality places on the observations made in the lab.

Most lasers are operated such that lasing occurs solely in the 0,0 transverse mode because it is this mode where the highest intensity is achieved. However, in any real beam, several of the higher order modes do appear to some degree. This, in addition to asymmetries in mirrors or other optical components in the laser cavity, results in a beam that has a minimum spot size that is some factor “$M$” times larger than the spot size of a diffraction limited pure Gaussian beam of the same wavelength.[1] The divergence angle will be this same factor $M$ times larger than that of a Gaussian beam. The parameter is generally expressed as $M^2$, since at any given place, the beam’s cross sectional area will be a factor of $M^2$ greater than that of a pure Gaussian beam and its intensity will be a factor of $M^2$ less. If the beam behaves differently along
two orthogonal axes, it may be instructive to calculate $M$ separately along these two axes. In this case, the factor describing the difference in intensity and spot area is $M_x \times M_y$. Using this definition, and the equation for the diffraction limited divergence angle, one can obtain an expression that can be solved for $M$, given the wavelength, the divergence angle of a focused beam ($\theta$), and the $1/e$ radius of the electric field at the minimum spot size ($w_0$).

$$\frac{\theta}{2} = \frac{\lambda M^2}{\pi w_0}$$

(1)

The objective of this paper is to describe a procedure by which $\theta$ and $w_0$ can be measured accurately, thereby obtaining a value of $M$.

**Method and Results**

In order to make an $M^2$ measurement, it is necessary to make width measurements of a focused beam [fig. 1]. Near the beam waist, these measurements yield a value for $w_0$, and in the far field, after the beam spread has become linear, they give the divergence angle. The laser used was a mode-locked Ti-Sapphire laser with a wavelength of 800 nm., a pulse duration of approximately 120 fs., a pulse frequency of 1000 Hz, and was generally run at about 790 mW throughout the experiment. The beam was focused initially using a 250 mm. focal length lens. Another lens of 200 mm focal length was used to image this focal region onto a CCD camera, providing a magnification of slightly more than x10. By moving the first lens on a translation stage, the plane that is imaged onto the camera is moved through the focal point. In addition, it was necessary to attenuate the beam greatly to achieve a reasonable intensity on the camera. First, the beam was reflected by a

![Fig. 1: Experimental Setup for M² Measurement](image)
100% reflectivity coated glass mirror. The light that bled through was then attenuated by several neutral density filters, of total ND 11.3, placed over the camera. The fact that so little of the beam energy is needed means that one need only pick off only a small part of the beam while the main beam remains in use in another application.

Initially, the camera used was an 8-bit CCD camera, the Watec WAT-902A. The beam was observed qualitatively to have separate behaviors in the horizontal (x), and vertical(y) directions. The width of the pulses was measured by using frame capture software to sum the image data independently in the x and y directions. Each of these two traces was then fit to a Gaussian, and the parameter corresponding to the $1/e^2$ radius of the Gaussian was recorded as the width.

This approach proved to be inadequate, as it could not obtain an accurate beam width. The largest problem was that of the baseline of the camera. The baseline was high enough to cut off the low intensity wings of the beam, discarding important information [Fig. 2]. In theory, if the beam were a perfect Gaussian, the fit would be the same regardless of the intensity cutoff, but since the fit only worked marginally well, it could not give accurate information about the behavior of the beam under the baseline. The fact that the Gaussian fit became especially inaccurate far from the focus of the beam doomed this approach completely. It may be possible to take accurate beam width data using an 8-bit camera, but good baseline control is a requirement.
The solution was to use a better CCD camera, the AT200: CH270, from Photometrics. This camera has a 16 bit dynamic range, a variable shutter, and has a 1024 x 256 pixel array that can be cooled to -127 degrees with liquid nitrogen to reduce dark current. The ability to see the entire beam now allows for the use of a better method than a Gaussian fit to measure beam width. This method uses the standard known as the second moment of the beam defined\[2]:

\[
(\sigma(z))^2 = \frac{\iint r^2(E(r,\theta,z))^2 \, da}{\iint (E(r,\theta,z))^2 \, da}
\]

(2)

\[
(\sigma_x(z))^2 = \frac{\iint x^2(E(r,\theta,z))^2 \, da}{\iint (E(r,\theta,z))^2 \, da} \quad (3a)
\]

\[
(\sigma_y(z))^2 = \frac{\iint y^2(E(r,\theta,z))^2 \, da}{\iint (E(r,\theta,z))^2 \, da} \quad (3b)
\]

Where the beam is propagating in the z direction and the centroid of the beam is defined to be at the origin. This definition places the x and y second moments at half the 1/e\textsuperscript{2} radius of the intensity for a Gaussian beam, though other definitions can be found that multiply this value by two or four\[1\]. Using the second moment, it is much less of a problem that the intensity profiles do not always fit well to Gaussians. However, the treatment of the signal far from the centroid now becomes critical, as these values have additional weight in the width calculation.

First, it was necessary to correct for any bias present in the camera. This was done by subtracting an image with the beam blocked for each image that was taken of the beam. The first attempted solution to the noise problem was to improve the signal to noise ratio by summing several frames of data. The signal level sums as the number of frames, while the level of the Gaussian noise sums as roughly the square root of this number. However, even after summing twenty frames of data, there was still enough noise that it was necessary to truncate the data at some very low intensity level. The second moment measurement was fairly sensitive to the exact level of this cutoff, a sure indication that an accurate measurement was not being made. In this case, it was not practical to increase the number of frames per image because of the time involved in taking the data. Since casual investigations suggested that the beam width of this particular laser system may change by as much as 8% over the course of a day, there is some limit on the time range in which the images used in a given M\textsuperscript{2} measurement should be taken.

A different method for preventing noise in the wings from corrupting the width measurement was to set all of the signal to zero outside of some fixed radius. To do this, a Gaussian fit was applied to the x and y direction. All data outside twice the 1/e\textsuperscript{2} radius obtained by this method was set to zero. Spiricon engineers have determined this distance to be approximately the range at which the useful information obtained by including more signal is overcome by the detrimental effect of including more noise\[1\]. For a true Gaussian, this process would only effect the second moment by about 0.27%.
In conjunction with this change, only two frames of signal were summed for each measurement.

In order to be sure that noise within the $1/e^2$ aperture was not impairing the width measurement, a simulation was done where a Gaussian pulse was generated with an amount of noise added so as to match the signal/noise ratio of the image data. The error in the computed second moments was negligible. This is because of the high dynamic range and low noise level of the camera used. An 8 or 12 bit camera with higher noise levels would have to sum many more frames of data to get comparable results.

For each image, the exposure time was set such the camera would be just under saturation at peak beam intensity. This was done in order to get the best possible dynamic range. A test done by taking multiple images of the same part of the beam with different exposure times verified that the width measurement was, at the very least, not adversely affected.

A summary of the final process is as follows:

1) Using the MAPS Measurement camera software, two 16 bit frames of noise are subtracted from two frames of signal. This arithmetic is done in a 32 bit array. The maximum and minimum values of the 32 bit data are recorded.

2) The data is then scaled and exported to Igor Pro data manipulation software as a 16 bit image.

3) All of the data in the image under the noise level is truncated. Since the scaling sets the lowest negative noise component to 0 in the 32 bit to 16 bit scaling process, the data now has an offset equal to the noise level, defined here as half of the peak to peak noise amplitude. This level is subtracted from all data, and all data under this level after the subtraction is set to zero.

4) Next, a procedure is run to determine the centroid of the intensity profile.

5) The image is then summed in the x and y directions and a Gaussian profile is fit to each trace.

6) Another procedure then eliminates all data outside the ellipse whose major and minor axis are formed by twice the $1/e^2$ radius in the x and y directions.

7) Finally, a procedure computes the second moment in the x and y directions, numerically evaluating equation 3 above.

8) The front lens on the table is advanced by an incremental amount, usually a millimeter or less, and the process is repeated.

A data run using this method is plotted in figure 3; notice that the beam focuses to a different size in the x and y directions and at a different place. In each direction, the $w_0$ measurement is made by applying a polynomial fit, 6th order in this case, to the data near the focal point. The divergence angle is determined by applying a linear fit to the data in the far field, roughly those points beyond the Rayleigh range. An $M^2$ value has not yet been calculated from this data.
Conclusion

While this method is believed to be effective, there are still many tests that should be run on it before a final judgment can be made. Before it can be considered valid, it must be able to achieve consistent $M^2$ measurements for a given laser, despite small changes in the optics used. Changes in the focal lengths of the lenses and in the amount of neutral density used could provide useful tests. Data should also be taken on a finer scale than the 1 mm. increments used on most test runs to date, especially near the focus. One way to improve the procedure would be to sum more frames of data per image. Creating a script to automate the image arithmetic could ease the time burden on the data taker, but the problem of maintaining beam stability over a data run would still have to be confronted. Also, it may be possible to improve the Gaussian fit used to determine a rough $1/e^2$ mark by making two separate fits along each axis, a different fit on each side of the peak. Tests would have to be run to see if this is worthwhile.

References


Fig. 3: Second Moment Beam Width Measurements Near the Focal Point