Granular Materials: Liquid-like Properties of Sand

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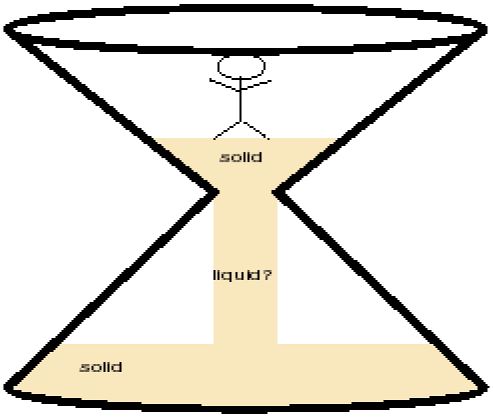
Introduction

Granular materials are made up of many small pieces that can move either separately or together. Some examples of different granular materials are sand, soil, grain, corn, plastic pellets, powders, and pills. These materials tend to have many different physical properties. They appear both liquid- or solid-like. For example, a pile of sand is more like a solid, while a vibrated or fluidized bed of sand can be more like a liquid like quicksand. This is important in life and industry.

There are many different situations that demonstrate this behavior. One example is an hourglass filled with sand. The sand can flow through the glass like a liquid, but at the same time a mass, such as a person, can stand on top of the sand as if it were solid (seen in figure 1). Another example is a sandcastle. With a sandcastle, the wet sand can be held together like a solid, but it can also fall apart as it dries, flowing down resembling an avalanche pattern. S. Nasuno, A. Kudrolli, *et al.*[1] have further described this stick-slip motion with sand. One of the main interests is the liquid-like property of sand during an earthquake. In an earthquake, the vibrations, with the help of water-saturated ground, the buildings to sink in the sand as shown in figure 2. P. Evesque[2] explained this property in his research. These vibrations make the sand act like a liquid. Though sand may appear liquid-like, it is not yet known if any standard fluid dynamical principles can be applied.

We would like to find some equations to describe the motion of sand since at the present there have not been any equations to describe this motion.

Fig 1



Hourglass

Fig 2

Sinking Buildings in Earthquake

We have chosen a granular system with very liquid-like properties. For instance, one of our main interests is working with sand as a fluidized bed. This is done by flowing air up through the bottom of the sand. With different stresses either by vibrations or airflow, the sand will become more solid-, liquid-, or gas-like. When the stress is low, the sand remains solid-like. Yet when the stress is high, the sand becomes liquid- and gas-like. There may be a range of stresses that make the sand behave as a simple liquid. Narayanan Menon and Douglas J. Durian[3] have found that at a low flow rate of air coming up through the sand using small sand granule size, the sand will become fluidized. Their main observation was that surprisingly even though you can easily stir the sand, the grains do not move when left untouched (but fluidized). P. Geldart and A. C. Y. Wong[4] have used different sizes of sand in their experiments. They found that 106 μm particles and smaller work best to make a fluidized bed. The sand appears fluid-like (easily stirred, surface stays horizontal) without gas bubbles. As you turn up the flow rate to fluidize the sand, the sand’s property changes from a solid, to a liquid where objects start to sink, then to a liquid with bubbles. We will try to concentrate on the middle regime.

There are many interesting things to study with a fluidized bed or with sand itself. Leo P. Kadanoff, O. Zik, *et al.*[5] have many categories in which sand is interesting to study. For example, in a fluidized bed sand undergoes convection where as the solid state of sand the top only moves. In our experiment, we have studied the fluidity in a fluidized bed of sand.

One of the main differences between a solid and a liquid is that in a solid there is an elastic constant where as a liquid has viscosity, where viscosity is the resistance to flow. A liquid cannot withstand shear forces so it cannot support a heavy object. A liquid wants to keep on flowing while a solid wants to spring back into its original position. What we were initially looking at is if sand in its fluidized state acts as a liquid with constant viscosity, otherwise known as a Newtonian liquid. If a force is applied to a sphere immersed in a Newtonian liquid, the sphere reacts with a drag force. This drag force is found to be directly proportional to the viscosity (η), velocity (v), and the size of the ball (a) for low speeds of the ball. This gives us drag force in a Newtonian liquid as $F_{Drag} = (6 \pi \eta v a)$, called the Stokes Drag. In our experiment, we want to find out whether the fluidized bed of sand obeys the same exact equation as a liquid. This means that holding everything constant, we would see whether the viscosity is a constant or a function of maybe it’s velocity, or even a function of grain size, object size, or gas flow speed.

The idea of our experiment is to find out viscosity's relationship to force by pulling a ball through the fluidized bed of sand. If you measure a, v, F_{Drag}, you can calculate η. A possible experiment is to drop a ball in a liquid and measure the velocity, v. When dropping the ball in a fluidized bed of sand, the ball can't be seen easily. In this case, in order to find the viscosity in sand, we chose to use a set velocity so we could measure the force that will in return give us the viscosity.

Experimental

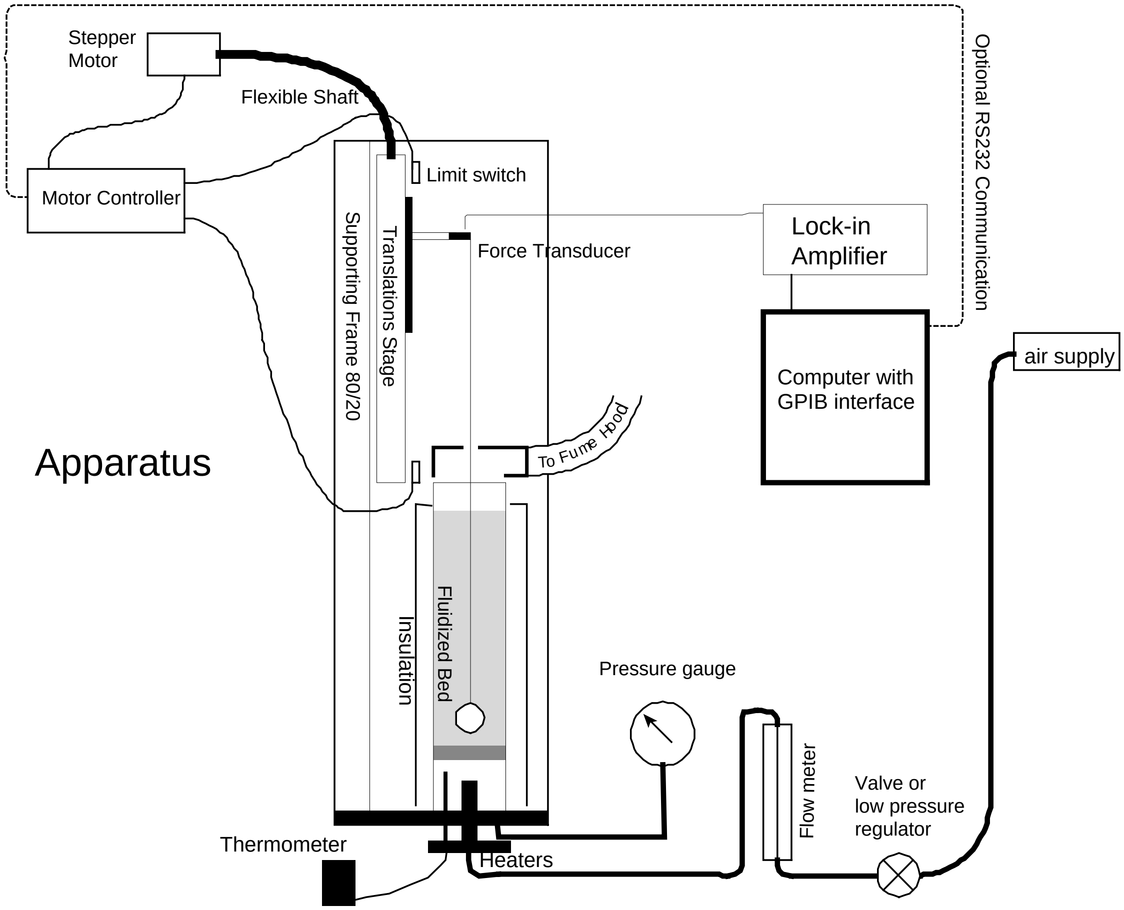
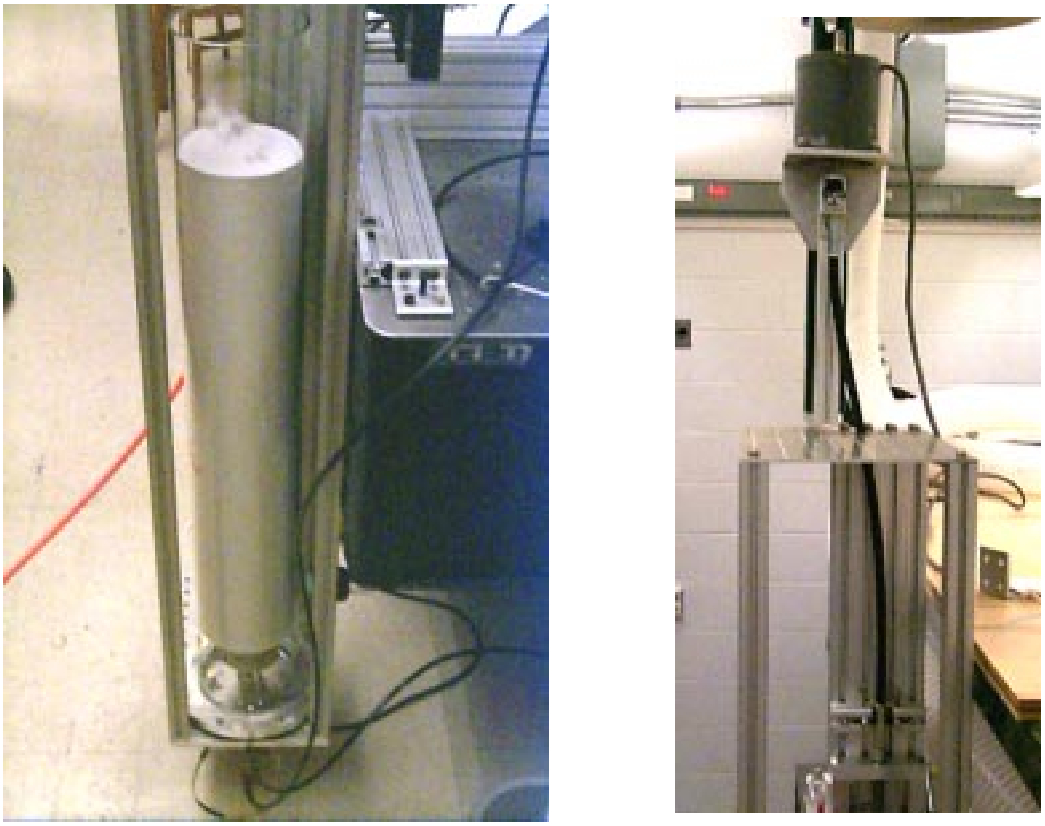


Fig 3a

The apparatus (shown in figure 3a and 3b) used in our experiment consists of a glass tube with a height of 36in and a diameter of 6in. A glass frit is built into the glass tube in order to hold the sand, yet allow the air to flow through the frit into the sand. Sand is put into the glass tube that are sphere glass beads sizes of 63-106 μm diameter. Air is pushed through the frit from a flow meter through a heater at 205 F to fluidize the sand at certain flow rates. The heater is used to drive off water vapor in order to prevent sticking or clumping. For example, in sandcastles, wet sand seems to use water as glue to hold the sand together, and as it dries the sand collapses into its many small pieces. This is why heating the sand is important. A ball is attached to a nylon wire (.01 in thick) that is then attached to a force transducer on a translation stage. The translation stage is connected to a stepper motor with a flexible shaft. The force transducer is attached to a Stanford Research Systems lock-in amplifier model SR830 DSP. The lock-in amplifier and motor controller are both connected to a computer that is programmed using LabVIEW. LabVIEW is programmed to move the translation stage up and down with certain velocities and receive data from the force transducer, which measures the force needed to drag the ball through the sand

Fig 3b



Fluidized Bed of Sand

Stepper Motor with Flexible Shaft

Force Transducer

Translation Stage

In order to find the viscosity of the fluidized sand, we set the velocity of the motor ranging from 0.06-1.5in/sec. We know the size of the ball that we used 2 different balls with diameters of 0.748in and 1.5 in. The force transducer is measured in µvolts. In this case, we must convert the force to Newtons. The calibration graph is show in figure 4. Also, the gas flow rate is measured by the location in mm that the ball rises on the flow meter. This must be converted into liters/m air (the conversion shown in figure 5) with the function of $y = -1.9252 + 41.8171x - 48.1052x^2$. This way we can calculate the viscosity and notice if it is a constant or function of velocity. To do this, we move the ball up and down the same distance and at the same velocity many times. The LabVIEW program takes the average of the data each time to smooth fluctuations in the measured force. To figure out the measured point from that graph, we subtracted the downward force trace from the upward trace to obtain the actual average force at that velocity and flow speed and divide by two.

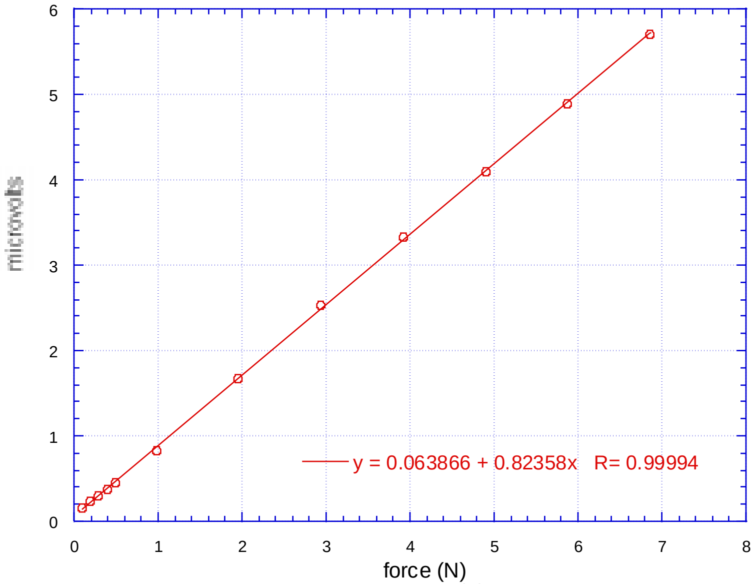


Fig 4

This calibration graph shows the conversion of force measured in millivolts to Newtons.

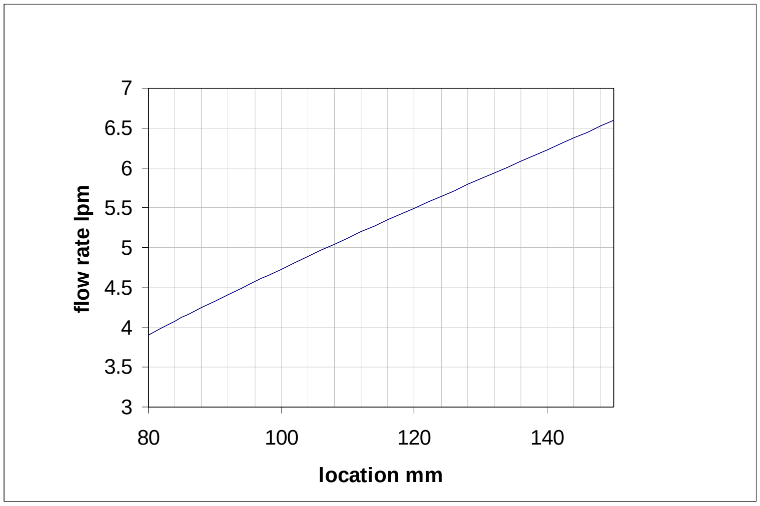


Fig 5

This graph shows the conversion for the flow rate from location in mm to flow rate in liters/min.

We found out that for different flow rates, the distance that the ball sunk differed (shown in figure 6). At the higher flow rate, the sphere was able to sink all the way down to the bottom of the sand, while at a lower flow rate the sphere could only sink part way in the sand. This was conducted by changing the gas flow and measuring the distance the ball travels without getting stuck (the string becomes limp). From this, we determined that the flow rate must be above 120 to expect a liquid-like behavior.

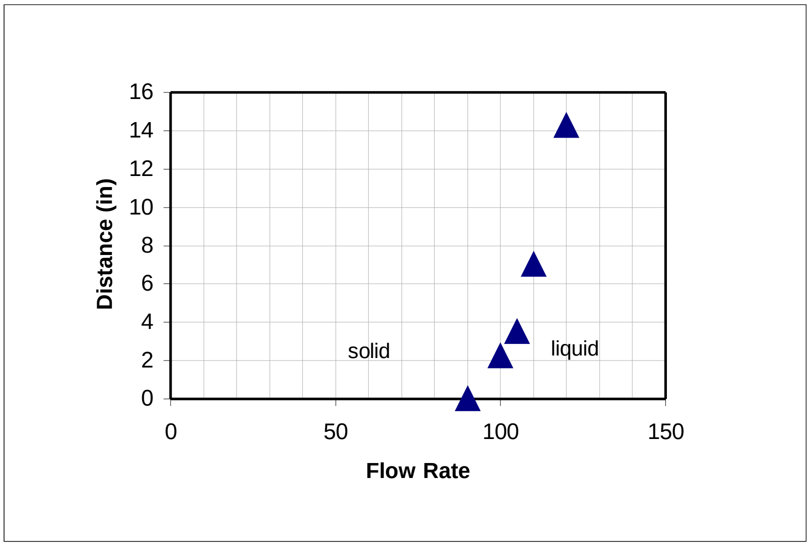


Fig 6

This graph shows that at different flow rates, the ball was able to sink at different distances.

From a set of data points plotted on a graph with velocity versus force at a constant flow rate of 120 (shown in figure 7), we can see that the plot of force vs. velocity is not linear. This means that liquid-like property of sand is not a simple liquid or Newtonian liquid. Not knowing which theory to apply, we arbitrarily fit a logarithmic function to the data. This is shown in figure 8. The logarithmic functional form also fits data taken at other gas flow rates, within some multiplicative factor. This graph also shows that at the lower gas flow rates, the drag force is larger than at the higher flow rates.

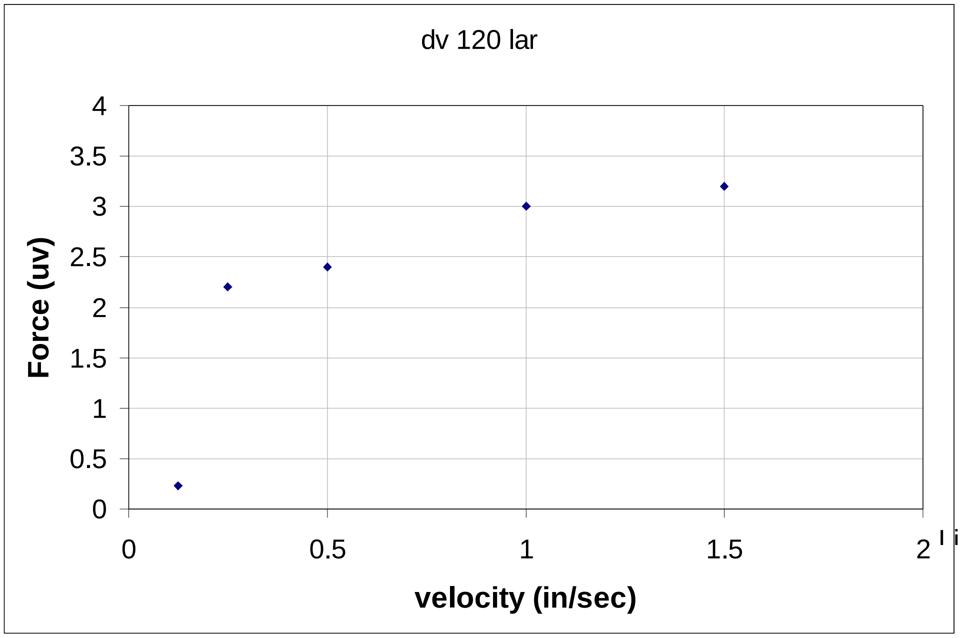


Fig 7

This graph shows the force versus velocity using a ball size of 1.5 in. The data is not obviously a linear line. Instead, with some trial and error with different functions, we saw that a logarithmic function was the best bet.

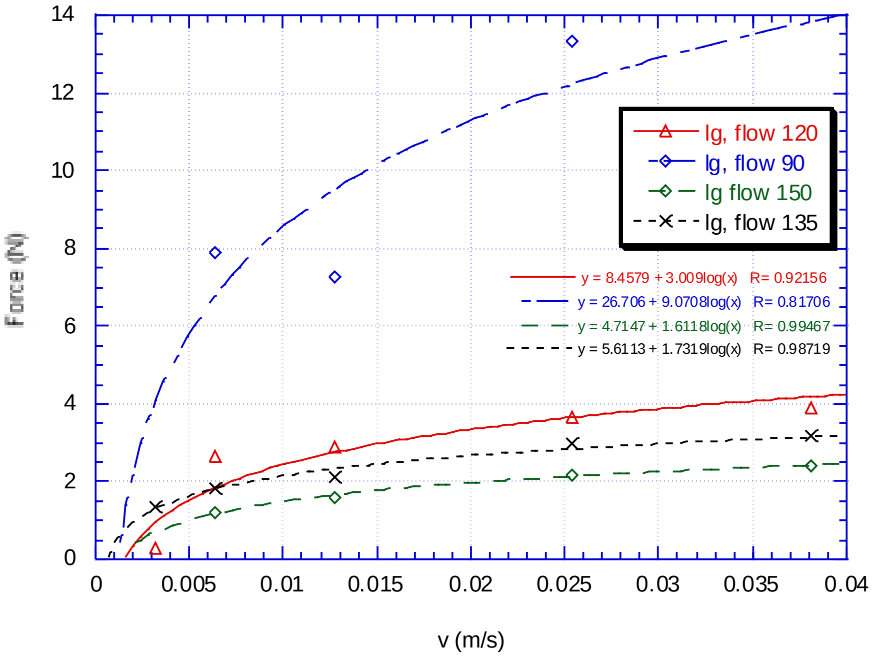


Fig 8

This graph shows at all the different flow rates, a logarithmic function first the data. This is good news, though we are still unsure what this logarithmic function means. Also, at lower gas flow rates, the force is larger than at higher flow rates.

Since this logarithmic function fits data for many different flow rates, we want to investigate if there is a systematic relationship for the spacing between these curves. Taking one velocity from the plot in figure 8, we can make a plot of force vs. the gas flow rate. This gives us a plot (figure 9) that diverges to a certain critical gas flow rate. Using the velocity of 0.038 m/sec, the plot (seen in figure 9) reaches a critical flow rate of 78. This plot gives us an equation, η (W) = 2.3 ((W-Wcrit)/Wcrit) $^{-1}$, where W is the gas flow rate and Wcrit is the critical gas flow rate. This approach was inspired by similarities of figure 10 to that of heat capacity near a second order phase transition. The critical flow rate of 78 is close to the point where we measured the sand to become a solid in figure 6. By looking at this plot (figure 10), it shows how well the sand becomes fluidized with different gas flows. The line seems to be a phase transition line where to the left of this line it is a solid and to the right it is a liquid.

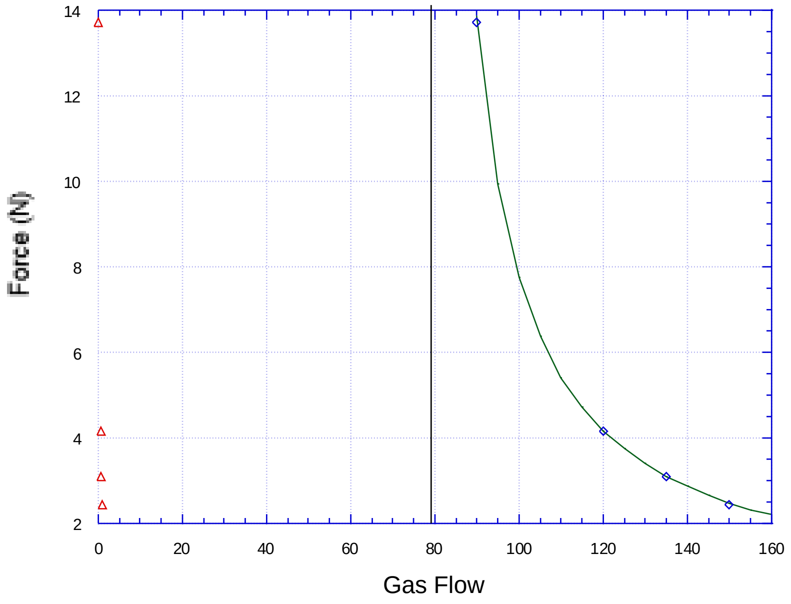


Fig 9

Above is the plot of Force vs. Flow rate at a constant ball speed of 0.38 m/sec. The line tends to diverge to a critical gas flow rate of 78. This plot resembles one of a second order phase transition.

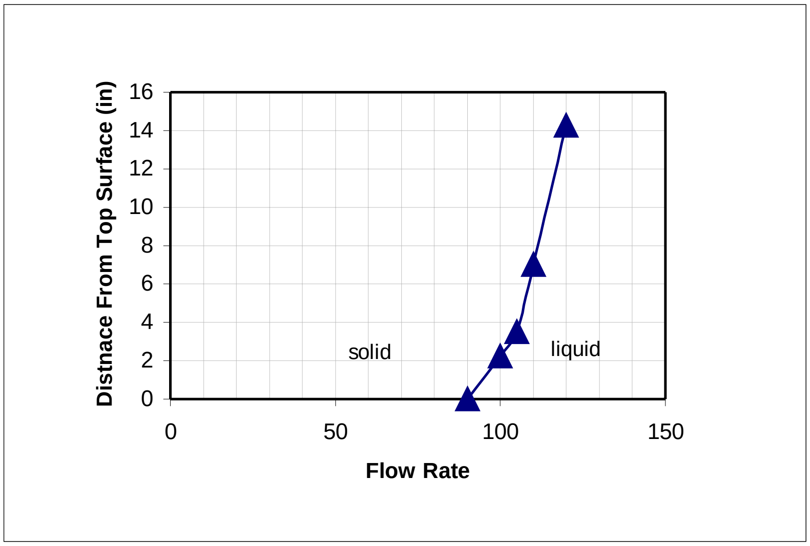


Fig 10

This graph is distance vs. flow rate. This shows how far the ball has sunk at different flow rates. This graph has the same critical flow rate at about 90 as in figure 7. The graph shows the phase transition line where the sand is a solid or a liquid.

The data lead us to a probable theoretical conclusion. Knowing that the function for the viscous force is 2.3 $((W-Wcrit)/Wcrit)^{-1}$, the drag force factors into the product of two equations. The drag force is equal to a function of velocity times the viscous force function of the flow rate. The calculated function turns out to be

$$F(v,w) = (1.9 + 0.064 \log(v)) \quad 2.3\,((W-78)/78)^{-1}$$

where F is a function of velocity and flow rate. We are unsure about the log part of this equation, but the viscous function of the flow rate seems to agree with our data quite well. This allows us to cover all the data points with this single function of velocity and gas flow rate. (shown in figure 11).

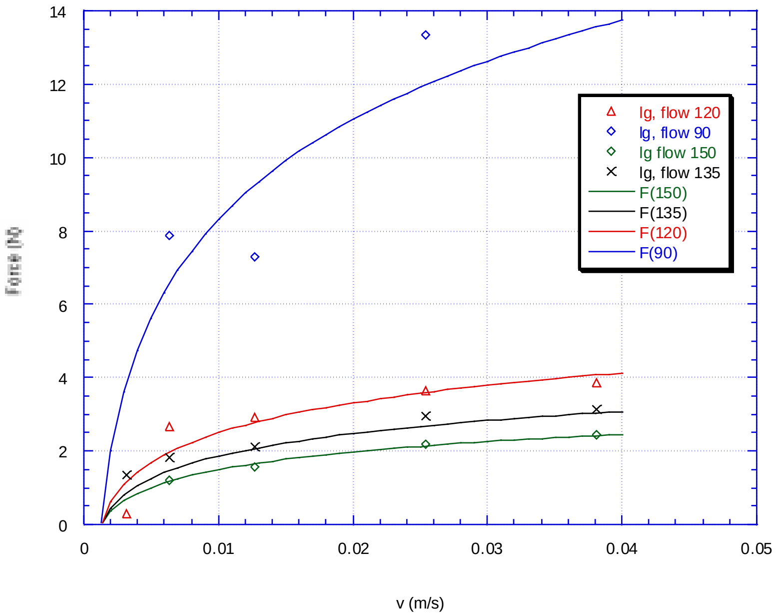


Fig 11

Above are the fitted lines to the data.

During the experiment we have done many different things to the apparatus to improve the data during the experiment. One major problem that we fixed was that the first frit was not allowing the sand to become fluidized evenly. We also had some problems that were fixed with the translation stage becoming stuck from sand getting under it. Also, the motor shaft was vibrating making a lot of noise in the data and the sand was not heating very fast. These things were repaired or corrected.

Conclusion

This experiment has allowed us to learn many different things about the fluidization of sand. We have learned that drag force can be described as the products of 2 functions, one of velocity, and one of the gas flow rate, $F_{drag} = g(v)\,\eta(w)$. We have learned that the apparent power law rate of is about $2.3\,((W-78)/78)^{-1}$. The drag force is a function of this function of viscosity and another function of 1.9 + 0.064 log (v). As of yet, we have no theoretical footing for the long function. The viscous function, however, seems to fit nicely with our data and is inspired by the theory of a second-degree phase transition. I have learned that experimental physics takes a lot of time to accomplish what we want with some trial and error and some educated guessing. As always, in order to learn everything that we want to, more time will be needed. Starting with one experiment has led to another exciting wonder of the fluidized bed of sand. There are still so many things to be studied about this new topic in the world of physics. I hope to study and learn more about it in the future.

Acknowledgements

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[1] Nasuno, S. and A. Kudrolli, *et al.* "Time-resolved Studies of Stick-slip Friction in Sheared Granular Materials." Physical Review E. Vol. 58: 2161-2171, 1998.

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