Two-Layer Raleigh-Benard Convection

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Abstract:

Interest in two-layer systems is inspired by its experimental accessibility, theoretical accessibility and its application in mantle convection and encapsulated crystal growth. Theoretical studies have predicted time-dependent patterns at or near convective onset. Studies[4] have also been performed in particular for a three dimensional system of silicone over Flourinert™ and silicone over water. There is hope that these systems will exhibit time-dependence as predicted. A functional test cell has been built, but extensive experiments have yet to be performed.

Introduction

Two-layer convection systems can be found in atmospheric physics, geophysics, astrophysics, or anytime there is a stratification of fluids. Interest[3] in the two-layer Rayleigh-Benard system is inspired by its applications in modeling convection in the earth's mantle, and solidification of crystals surrounded by fluid. As well as being accessible experimentally, the system readily presents itself to theoretical analysis. Some systems must be driven far into non-linearity before exhibiting time-dependence. Since the system can exhibit time-dependence near convective onset, weakly non-linear equations of motion can be used to analyze the behavior. Experiments[1,3] have been performed with a quasi-two dimensional cell involving two immiscible fluids. A thermal gradient was imposed on the system, and the convection patterns were visualized using a shadowgraph technique. Time-dependence was found for the silicone over water system in the rectangular cell. Time-dependence was found for both the silicone over Flourinert™ and the silicone over water system in the annular cell. My project involved extending the narrow dimension of the rectangular experiment to investigate the possibilities of the unexplored three-dimensional case. The bulk of my project focused on the design and construction of 3.1" x 4.5" x 0.5" cell and the system needed to test it. Little time was left for taking data on the system, but we did achieve imaging for the one and two-layer cases.

Numerical Studies

Interesting patterns can arise beyond standing wave patterns. Traveling waves and patterns that neither travel nor stand but oscillate are found. From the theoretical[4] side, silicone over water is more prone to time-dependent pattern formation at onset since a wider range of depth ratios allow for this behavior. Future experiments will likely involve this combination. The following image was taken from D. D. Joseph et al in **Fundamentals of Two-Fluid Dynamics**. It represents an oscillating pattern when viewed from above the cell with the thermal gradient normal to the page. The colors indicate varying degrees of a dimensionless vertical component of the fluid velocity.

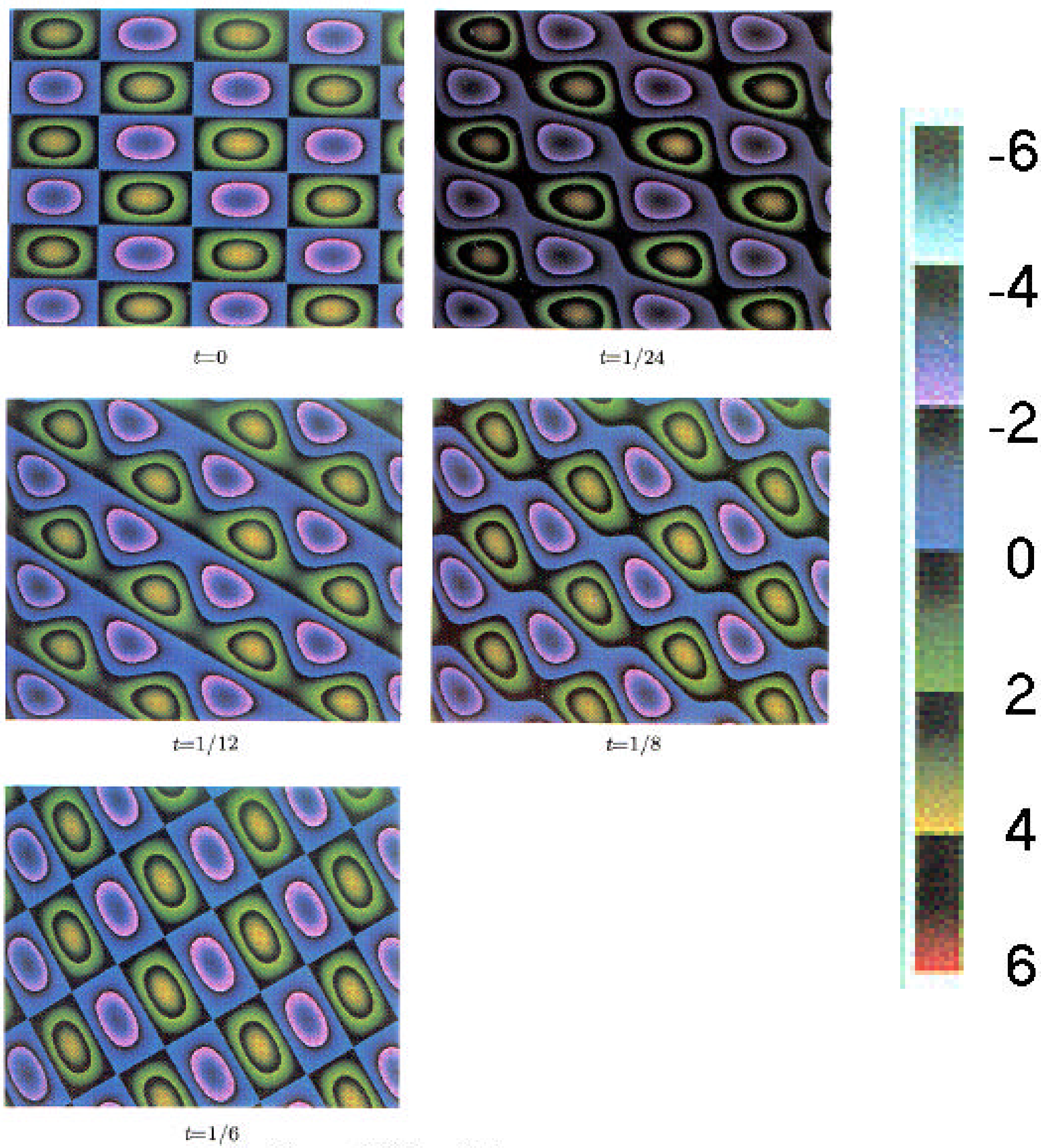


Figure 1: Wavy Rolls

Design

The three dimensional cell poses the problem of visualization. The cell is visualized from above, so light must pass through the cooling chamber into the cell and back out. It must therefore have a lower plate that is both highly thermally conductive, and reflective enough for the shadowgraph technique. One must also have a thermally conductive plate across the top of the cell that will transmit light for visualization. It was proposed that we satisfy the first requirement with a thick, polished, copper plate. The second requirement was fulfilled with a ¼" thick, clear sapphire plate. The rest of the process was a matter of finding appropriate construction techniques and dealing with other design details along the way. Figure 2 below shows a cross-section of the cell system indicating the various components. Figure 3 is a view into the cooling chamber with the polished copper visible.

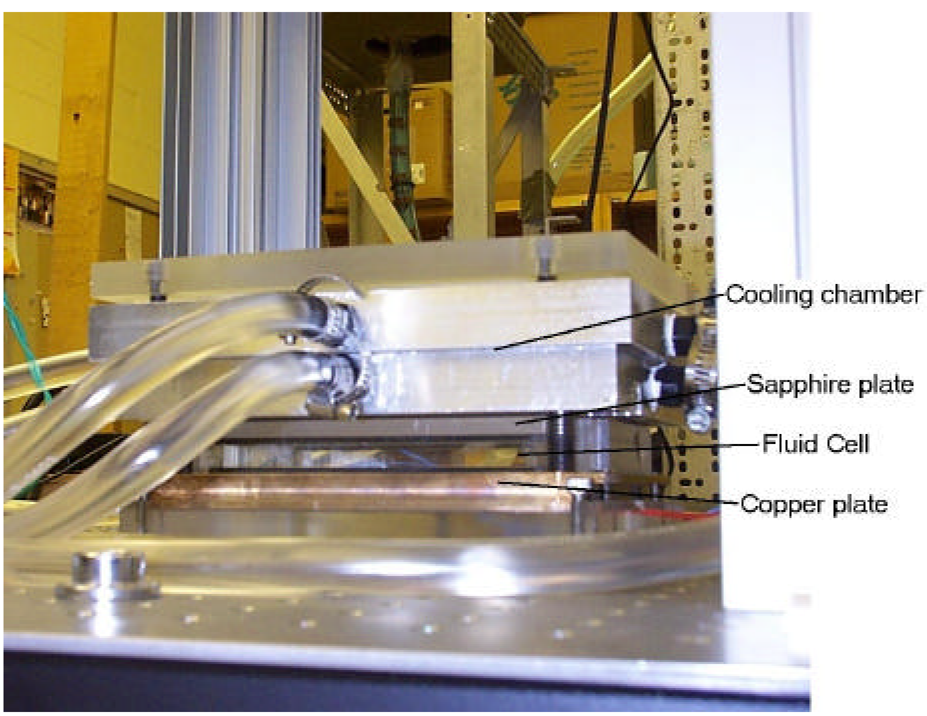


Figure 2

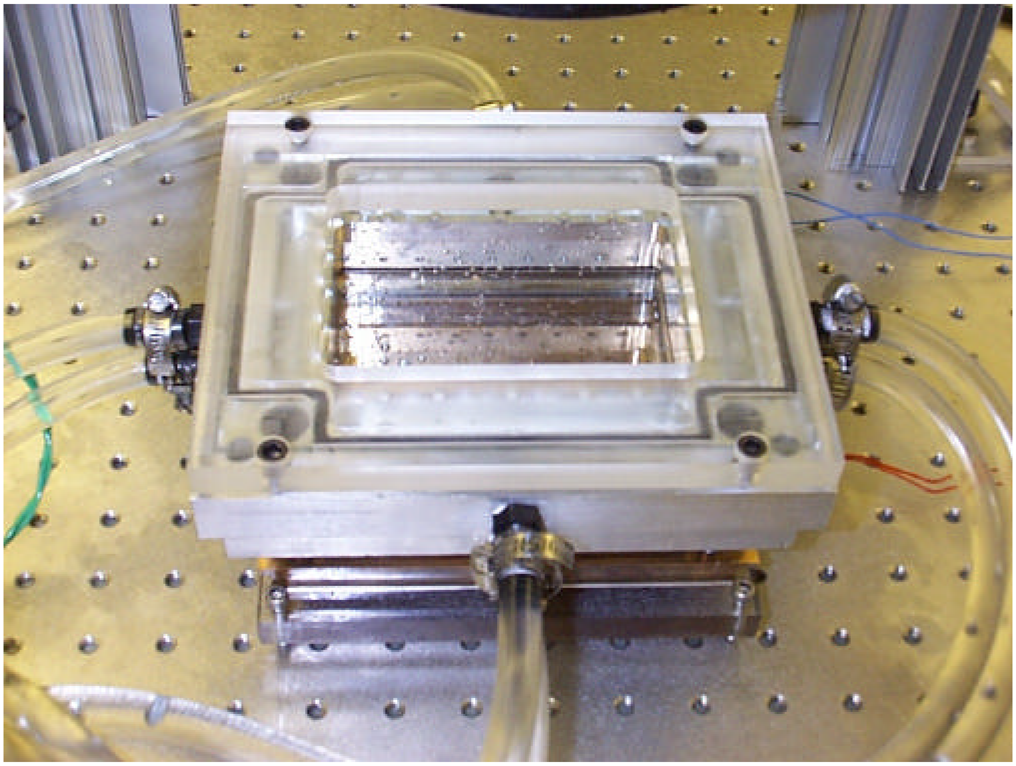
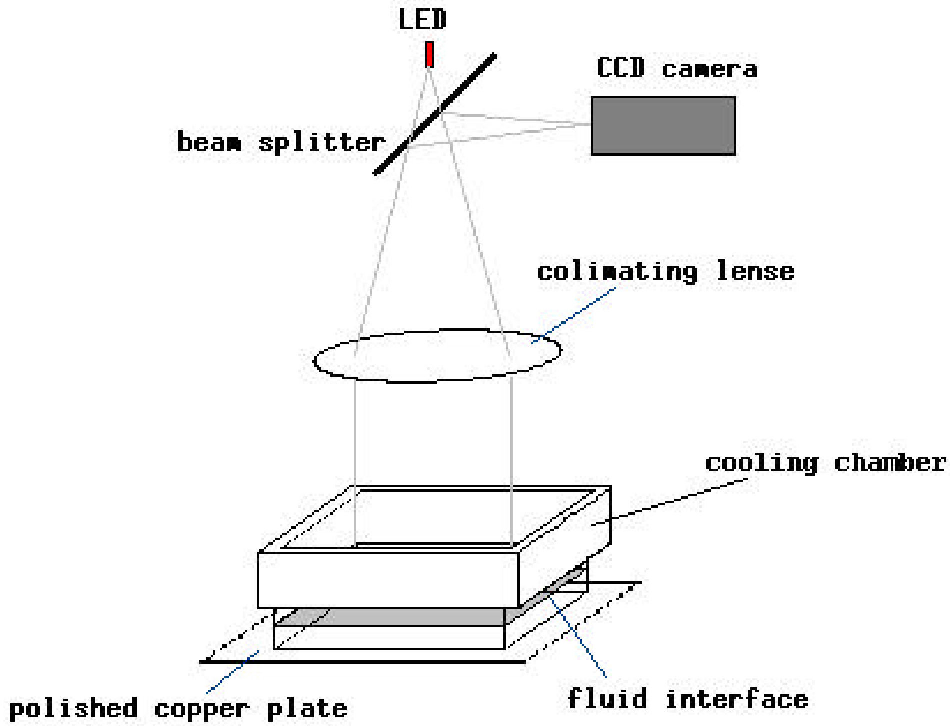


Figure 3

Experimental Setup

The shadowgraph technique is employed for visualization. The optical system is mounted in a tower-like frame above the cell system. This includes a collimating lens, a beam splitter, an LED and a CCD camera (see schematic 1 and figure 4). The fluids we use have a sensitive index of refraction. The colder areas will converge the light and produce a bright image while the warmer areas will tend to scatter the light and produce a dark image. It is a highly sensitive optical technique and convection patterns can be detected with thermal gradients across the fluids as low as 0.6 C°. Of course, images retrieved at this difference require background subtraction to see clearly, but images at as little as $\Delta T = 1.2$ C° can be seen clearly on the monitor.



Schematic 1

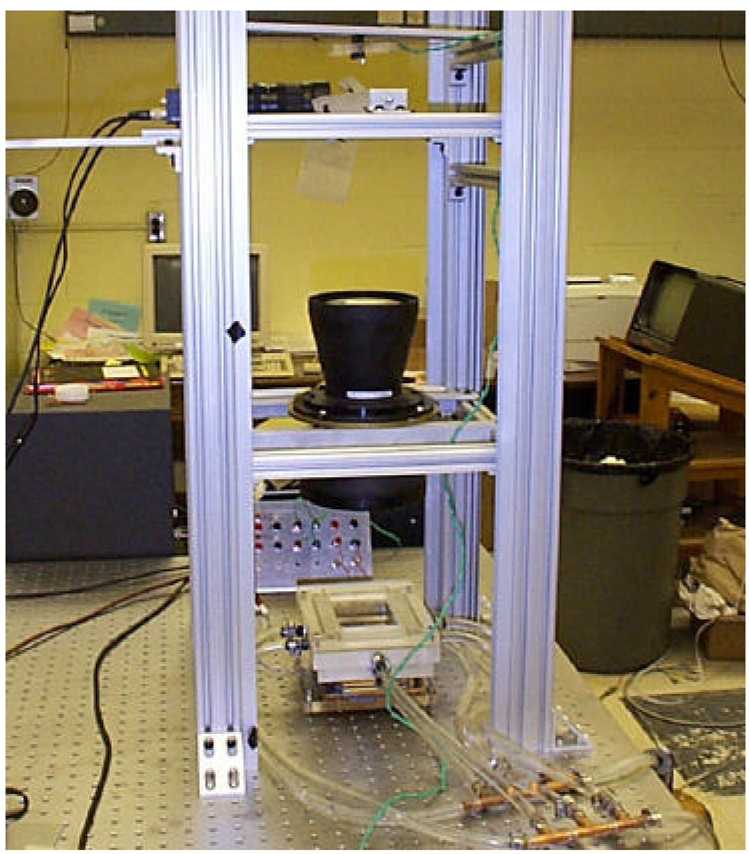


Figure 4

Early Images

Bitmap images were taken with a computer for visualization. Some images were processed using background subtraction, amplification, and a smoothing function on LabView™. These images where achieved while testing the system. They do not represent controlled experimental results.

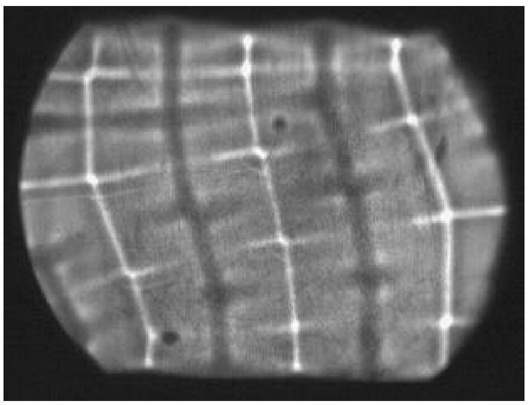
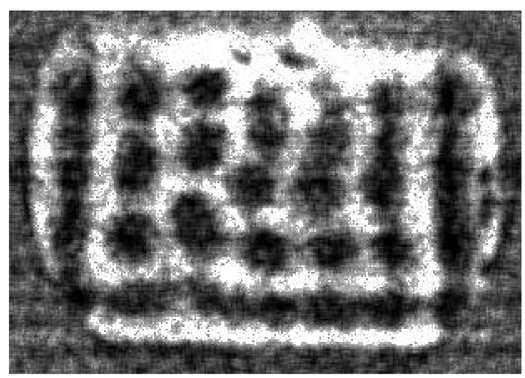


Figure 5: one layer silicone forming rectangular steady state

Figure 6: background subtracted two-layer silicone over Flourinert forming circular cells.

T=0.7 C



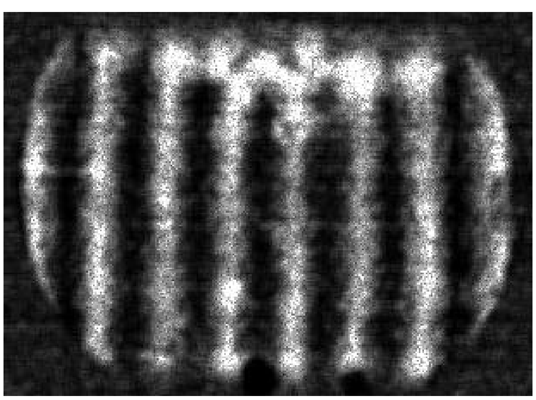


Figure 7: background subtracted Two-layer silicone over Flourinert Forming steady state rolls

T=0.8 C

Conclusion

Construction of a functional test cell has been achieved. Some improvements have yet to be made. There is a significant leak in the actual cell that must be fixed. A larger collimating lens is needed to visualize the entire cell. The system needs to be mounted on a floating table due to the sensitivity of the optics to distortions in the fluid interface. A thermistor needs to be mounted somewhere in the cooling chamber to achieve a fine reading of the upper temperature. Also, if the silicone over water system is to be pursued the copper must be plated or sealed against any water coming into contact with it due to its highly corrosive nature. Ultimately, this work has opened the door to further investigations of this system.

Acknowledgements

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[1] C. D. Andereck, P. W. Colovas, M. M. Degen, and Y. Y. Renardy, Int. J. Eng. Sci. **36**, 1451 (1998).
[2] D. D. Joseph and T. T. Renardy, *Fundamentals of Two-Fluid Dynamics* (Springer Verlag New York, 1993).
[3] M. M. Degen, P. W. Colovas, C. D. Andereck, Phys. Rev. E. **57**, 6647 (1998).
[4] Y. Y. Renardy and C. G. Stoltz, *Time-dependent pattern formation for convection in two layers of immiscible liquids*, (1999).