Investigating the Small-Gap Limit in Taylor Couette Flow

Daniel Arking
University of Chicago, 2005

In the Lab of Dr. C. D. Andereck
The Ohio State University
Smith 3021

OSU REU
Summer 2004
Werner Heisenberg was asked what he would ask God, given the opportunity. His reply was: “When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first.”

We have written the equations of water flow. From experiment, we find a set of concepts and approximations to use to discuss the solution – vortex streets, turbulent wakes, boundary layers. When we have similar equations in a less familiar situation, and one for which we cannot yet experiment, we try to solve the equations in a primitive, halting, and confused way to try to determine what new qualitative features may come out, or what new qualitative forms are a consequence of the equations.…

The next great era of awakening of human intellect may well produce a method of understanding the qualitative content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrodinger’s equation contains frogs, musical composers, or morality – or whether it does not. We cannot say whether something beyond it like God is needed, or not. And so we can all hold strong opinions either way.

---

1 From www.wikipedia.com
2 Feynman, Richard, Feynman Lectures on Physics, Vol. 2, 41-12
Introduction

We are surrounded by fluid bodies. From tides in the ocean to the air under an airplane’s wing to ripples in a bath tub, we spend our lives literally surrounded by fluid motion. And despite its commonness in the everyday macroscopic world, the physical equations describing such motion remain beyond our ability to solve. This, however, has not stopped us from trying. This paper seeks to add to the effort of understanding fluid motion more completely by reviewing my work on the flow created in the small gap limit of the Taylor Couette flow. I review some of the mathematical foundation for fluid dynamics, present my experiments and the observations that I have drawn from them, and hopefully provide some insight on this system’s role in continuing fundamental fluid dynamics research. Finally, this paper is at least in part a scientific paper, but it is also a review of my work this summer, and what I have learned from my research, not just what I have accomplished in the lab.

A Little Background

The foundation for fluid dynamics is navier-stokes differential equation. It is given as:

\[
\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{\rho} + \nu \nabla^2 u + \frac{\mathcal{F}}{\rho},
\]

Where \( P \) is pressure, \( \rho \) is density, \( u \) is the velocity vector field, and \( \nu \) is the kinematic viscosity. While the navier-stokes equation can not be solved analytically, it can be solved computationally. In the case of Taylor Couette flow, we also have the boundary conditions:

\[
\begin{align*}
U(r_i) &= r_i \ast \Omega_i \\
U(r_o) &= r_o \ast \Omega_o \\
U(z = 0) &= r \ast \Omega_o
\end{align*}
\]

These are known as no-slip boundary conditions. In words, they state that at the boundary between a fluid and a solid, the fluid and solid move together with zero relative velocity\(^4\).

---

\(^3\) From http://scienceworld.wolfram.com/physics/Navier-StokesEquations.html
Using these equations, we can develop a parameter known as the Reynolds number (R). For Taylor Couette flow in cylindrical coordinates, it is given by:

\[ R = \frac{d \cdot r \cdot \Omega}{\nu} \]

where \( d \) is the gap width, \( r \) is the radius of the cylindrical boundary, \( \Omega \) is the angular velocity, and \( \nu \) is the kinematic viscosity. Since both cylinders can move independently, we need separate values for inner and outer Reynolds numbers (Ri and Ro, respectively). Using the Reynolds number instead of \( d, r, \Omega, \) and \( \nu \) as independent variables allows us to plot flows in 2-dimensional parameter space what would otherwise be a mess. For example, Andereck et al. developed a parameter space map of a typical Taylor Couette flow:\(^{5}\)

---

**The Taylor Couette System**

\(^{4}\) This is a small point, but I am always amazed by the notion that no matter how smooth a solid surface is or how fast it is moving, whether it's a boat hull or plane wing, at least a very thin layer of the surrounding fluid will travel along at the same speed. Very cool.

The Taylor Couette System consists of two nested concentric cylinders. They are separated by a gap of constant width. Each cylinder is independently mounted and driven by separate electric step motors that can be controlled by computer to a precision of 0.001 Hz. In the system used, the fluid had a free upper boundary. At the bottom of the cylinders, the fluid filled the space between the bottoms of the two cylinders; however this region was effectively isolated from the cylindrical gap with a rubber gasket.

In a typical Taylor Couette system, the ratio of the inner to outer radius, $\eta$, is in the range of 0.7 - 0.8. In the small gap limit however, $\eta$ is as close to 1 as possible. The cylinder radii of the system used were 29.84 and 29.54 cm, making $\eta = 0.99$. Thus, this system is an excellent one for testing the small gap limit. This limit is interesting because as the gap gets smaller and smaller, the relative curvature within a given azimuthal length decreases and approaches plane Couette flow, which is created by moving two parallel flat planes relative to each other. Each system exhibits unique flow patterns, and for different physical reasons, however the small gap limit can be used to uncover any similarities.

Questions

I sought to answer three questions. First, how are the Taylor and plane Couette systems similar in the small gap limit? This is primarily a qualitative question, and one I investigated by comparing visual observations of the flow in the lab with literature on the plane coquette system. Second, how does the turbulent nature of flow increase as a function of inner Reynolds number? Third, is the turbulent nature of flow at a given Reynolds number unique? In other words, is hysteresis present in a plot of turbulent flow as a function of inner Reynolds number?

Procedures and Data Analysis
Over the course of the summer I conducted +++ tests. Water was used initially, however I switched to a 20% glycerol solution. The glycerol allowed the kalliroscope flakes to remain in suspension longer. The kalliroscope flakes were added in a 2% concentration. At this concentration the flow becomes visible without any significant changes in viscosity. Before each test, the inner cylinder was removed and the whole system was cleaned to remove deteriorated kalliroscope flakes from the previous tests. An everyday glass cleaner with water was sufficient. After reassembly, the gap was filled to 1 cm below the top of the inner cylinder and set in motion to get out all air bubbles. The kalliroscope was added from the top with a syringe and applied evenly around the circumference. After running the inner cylinder at Ri ~ 1500 for a few minutes the kalliroscope mixes throughout the fluid.

Once the kalliroscope was added, the digital camera used for data analysis was adjusted and focused. Once this was completed, the parameters of that particular test were entered into one of the LabView\textsuperscript{6} programs that were used to coordinate the speed of the motors and the camera so that data was taken at precise intervals at predetermined Reynolds numbers. Different programs were used for different tests (turbulence, hysteresis, etc.). Once the parameters were entered and the program started, the computer automated the actual data acquisition; however I found it useful to make visual observations first hand in addition.

To allow visual observation of the flow, the outer cylinder is made of clear plexiglass. A digital video camera recorded the flow and saved the images to a computer. To accurately measure the average flow at a given Reynolds number, the camera recorded not two-dimensional still images, but one-dimensional vertical cross sections at a rate of 10 times per second. Labview then compiled these strips into a two-dimensional space-time diagram. Such a diagram contains information on the flow past a given point over a number of minutes. Figure 2 shows how the space-time diagrams are created:

\textsuperscript{6} Chris Carey, a former student in Prof. Andereck’s lab, designed and created the original programs that I later adapted and modified.
A Matlab program quantitatively analyzed each space-time diagram by calculating the second derivative of the light intensity fluctuation across each line of pixels. Pixels with a second derivative over a predetermined cutoff value were deemed “turbulent”. Finally, the program determined the percent of all pixels in the space-time diagram that were above this cutoff. This value is called the turbulence factor, and gave a quantitative measure of the turbulent nature of the flow.

In addition to the quantitative methods described above, I also spent a lot of time staring at the flow. This allowed me to get a relatively intimate feel for the fluid behavior and how it changes. I also made observations that the computer programs simply were not set up to record, such as how the flow changes from one regime to another, or how quickly the flow responds to changes of the inner Reynolds number. Most importantly, however, Taylor Couette flow just looks really cool. The flow is visually aesthetic in addition to scientifically interesting, and I found it very easy to spend long periods of time just sitting in the lab watching it.
Observations: The Flow

As the inner Reynolds number changes, the fluid experiences changes in the flow pattern. These changes are more or less continuous, but can be separated into a few discrete flow regimes. For all regimes described here, the outer Reynolds number (Ro) was 0. Between inner Reynolds numbers (Ri) 0 and 490, the flow is in the azimuthal direction a remains featureless. Above Ri = 490 the centrifugal forces on the fluid are high enough that radial currents develop. At the outer cylinder boundary these currents are deflected back inward and as a result numerous vortices form in the azimuthal direction. They are stacked on top of each other, and are continuous around the entire circumference of the gap. This is called Taylor vortex flow. At Ri ~ 500 the vortices begin to exhibit an amplitude modulation. This regime is known as wavy vortex flow.

Beginning at Ri ~ 520, local regions of short wavelength disturbances develop. These are known as very short wavelength bursts (VSWB) and occur within the overall wavy vortex flow. A single VSWB is typically a few square centimeters in area, and lasts for a duration of a few seconds. As the inner Reynolds number is increased, the characteristics of a single VSWB remains the same, but the density of these bursts increases. At higher values of Ri that were examined (Ri = 1100 - 1200), VSWB nearly fills the flow.

It is important to note that while the “turbulent factor” of the flow increases with Ri, the flow itself is not actually in a truly turbulent state. Turbulence is defined as a condition of chaotic motion at all distance scales. While VSWB, especially in higher densities, may mix the fluid in a seemingly chaotic nature; at smaller length scales the flow remains laminar. It is possible to create a thoroughly turbulent flow, however only at higher values of Ri than I tested.

Results

This section will review the results of the computer controlled tests that I conducted this summer. These tests focus on answering the second and third of the questions I stated above.
To test how the turbulent nature of Taylor Couette flow changes as a function of Ri, I set the LabView program that coordinates the motors and camera to run according to the following parameters:

- Outer Reynolds number = 0
- Initial Ri = 480
- Final Ri = 1200
- Reynolds increment = 10
- Time delay = 2 minutes
- Data acquisition time = 3 minutes

Initial tests showed that the turbulence factor of the flow increases until Ri ~ 900, when the turbulence factor plateaus at a constant value (70 – 80 %). VSWB appears to saturate the fluid. This can be seen in figure 3, which shows data taken on June 30, 2004.

![Figure 3: While the turbulence factor increases steadily with inner Reynolds number, it plateaus above Ri ~ 900. Turbulent flow appears to saturate the fluid.](image)

This test took a total of 260 minutes. However, when a longer, 22.4 hour test was conducted on July 12, 2004, a new curve was produced (figure 4):
This plot implies that as the fluid is driven at higher Reynolds numbers, the flow actually becomes smoother! This obviously makes no sense. Further testing showed that the turbulent nature of the flow was not decreasing, but it was becoming invisible to the camera. In other words, the kalliroscope was settling out of suspension.

I adapted the Labview program to run the motor at a constant speed, and did a long term test to see how the kalliroscope settles out of the flow at a constant Ri. The test was conducted at Ri = 1000 and run continuously for 62.5 hours. In addition, a solution of 20% glycerol was used instead of pure water. This increased the viscosity by 61% in hopes of keeping the kalliroscope in suspension as long as possible. The space-time diagrams were scanned as usual, and the following plot was produced (figure 5). The kalliroscope falls out of suspension linearly with time. All tests that I conducted afterwards were, as a result of this plot, limited to 5-6 hours (iterations 20 – 24).
To test for the presence of hysteresis, I further adapted the original Labview program to ramp up and then immediately back down again. This loop could also be repeated any number of times within a given test. The first of this type of test was conducted on August 5, 2004. It had the following characteristics:

Ro = 0
Initial Ri = 480
Final Ri = 1100
Reynolds increment = 10
Delay time = 2
Data acquisition time = 3
Number of loops = 2
Total trial duration = 10 hours, 20 minutes

The plot, shown in figure 6, shows that no hysteresis is present in the turbulent factor of the flow. It does, however, exhibit the effect of settling kalliroscope. Each loop was constrained to five hours, but over the total time, a relative decrease in the observed turbulence factor is apparent.
Figure 6: There is no sign of hysteresis in two consecutive loops between $R_i = 480 – 1100$. The decrease in turbulence at a given Reynolds number is due to deteriorating kalliroscope, not a physical property of Taylor Couette flow.

This shows that the turbulent factor of the flow is uniquely determined by the inner Reynolds number, as one would expect. Just from our everyday experiences, we expect a given body with a given agitation to have a given level of turbulence. This test confirms that intuition (which, I must say, is quite satisfying).

Conclusions and Insights

This spring I requested to work in a fluid dynamics lab because I could not turn on a faucet or put milk in my coffee without taking at least a second to marvel at the flows before me. After a summer studying Taylor Couette flow I take even longer at the sink and pour my milk a little slower, lest I smother the currents swirling around in my cup. I have learned many useful tools, including the LabView computer program, basic metal working techniques and of course how to rebuild a multi-ratio gear box (my dad’s 5-speed transmission is next). More
significantly, however, I have practiced how to take a curious phenomenon and methodically investigate it. To me that is the essence of the scientific method at work, and in this aspect the summer has been very rewarding. But I digress. Not all of my conclusions are philosophical; and my conclusions on the data I took deserve some attention.

First, the occurrence of instabilities such as Taylor vortex flow, wavy vortex flow and of course VSWB occur at predictable Reynolds numbers, and they can be created at these Reynolds numbers with high accuracy. It is intuitive that any fluid flow should become more turbulent as it is driven at higher speeds (controlling for viscosity, gap width, and inner and outer radii); however, it is interesting that the turbulence factor of the flow plateaus above Ri ~ 900. I believe that this results from the fact that while relatively turbulent, the flow is still laminar at small length scales, and thus turbulence can not fill these small spaces in the flow. To examine this, I suggest testing at much higher Reynolds numbers in which the flow is truly turbulent. Other experiments investigating turbulence in Taylor Couette flow look at Reynolds numbers between 1,000 and 1,000,000. My tests only grazed the lower boundary of this.

I was able to repeat the hysteresis test three times, and each time the data shows no presence of hysteresis. I have not seen any literature concerning the hysteresis in turbulent nature of Taylor Couette flow, and thus can not justify my results on theoretical grounds, but I still think the results are pretty cool.

While the data taken thus far provides useful information on Taylor Couette flow, there is lots of room for refinement. Smaller Reynolds increments, larger spans between initial and final Ri, and longer data acquisition times are all ways to improve the data quality. The main limitation remains the kalliroscope. Given its limited suspension, tests must balance the kalliroscope deterioration with the need to take good data. Using a 20% solution of glycerol instead of pure water improved the suspension time, however more still needs to be done before better data is possible. I do hope that the work will continue after the summer is over!

---

7 Lathrop, Fineberg and Swinney, Turbulent Flow between Concentric Rotating Cylinders at Large Reynolds Numbers, Phys. Rev. Vol. 68, 1515-1518