

The Effect of Warm Dark Matter on Primordial Helium Production

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Abstract

Successful on large scales, cold dark matter (CDM) theory has several problems at the galactic level. Warm dark matter (WDM) is intended to resolve these issues by having particles with initial velocities. We determine the change in the mass fraction of helium-4 produced during Big Bang nucleosynthesis by the introduction of a warm dark matter particle to the total energy density of the early universe.

Introduction

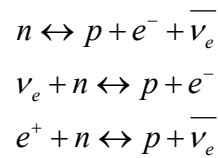
Dark Matter

In 1933, Fritz Zwicky determined from observations of the Coma cluster that the galaxies of the cluster possessed velocities greater than the escape velocity of the cluster. From this, Zwicky declared that more mass must be contained in the cluster than the observed visible matter¹. Thus began the history of dark matter. A number of different models have been proposed to explain dark matter. One proposed candidate is hot dark matter (HDM). HDM particles are relativistic with a low mass. HDM has been largely dismissed due to its lack of ability to predict large-scale structure correctly. The predominant candidate today is cold dark matter (CDM). CDM particles have very low velocities due to their mass. CDM successfully predicts large-scale structures, such as clusters, and the power spectrum of cosmic microwave anisotropies. However, CDM also predicts an overproduction of dwarf galaxies surrounding larger galaxies and cores of galactic halos that are denser than observed. These problems are addressed by warm dark matter (WDM)².

WDM causes damping of small-scale structure due to the thermal motion of WDM particles. This means that fewer dwarf galaxies are predicted and corrects the halo core problem. A number of WDM candidate particles have been proposed over the years, including thermally decoupled relics and particles formed by non-equilibrium decay processes, such as weakly interacting massive particles (WIMPs) like the neutralino, the least massive supersymmetric particle ³.

Big Bang Nucleosynthesis

The theory of big bang nucleosynthesis (BBN) started with George Gamow in 1946 ⁴. BBN states that most of the low mass elements in the universe were created soon after the big bang. At temperatures above 1 MeV, equilibrium between the number of protons and neutrons was maintained by the weak interactions ⁵:



At around $T=1$ MeV, the three neutrino species decoupled from the primordial plasma when the rate of neutrino interactions fell below the expansion rate of the universe. The neutrino temperature T_ν from this point would scale as the inverse of the scale factor (R^{-1}). As the temperature falls below 1 MeV, the rates of the weak reactions involving protons and neutrons fall below the expansion rate and the neutron-to-proton ratio freezes out at 1/6. The overabundance of protons is caused by the weak interactions favoring proton production due to the neutron having a slightly higher rest mass than the proton. This ratio is sensitive to the expansion rate and a change in the expansion rate will result in an increase or decrease in the number of neutrons produced. This is important because most neutrons will be incorporated into helium-4 during the process of nucleosynthesis.

When the universe is about 1-3 minutes old and the temperature is about 0.1 MeV, nucleosynthesis occurs. Due to neutron decay, the n/p ratio drops to about 1/7 at the time of BBN. As a result of a chain of reactions, most neutrons end up in helium-4, while others are locked into deuterium, lithium-7, or other elements that are produced in trace amounts⁵. In 1967, Wagoner, Fowler, and Hoyle wrote a code to predict to abundances of elements that are produced in BBN. The FORTRAN version of the code that Wagoner wrote in 1973 has been updated through the years and has become a standard for BBN study.

Goals

The aim of this project was to find the energy density of WDM particles at the time of the nucleosynthesis. This would be incorporated into the total energy density of relativistic particles in the Wagoner code. The result of this change would be an increase in the expansion rate, leading to an increase in the number of neutrons. As a result, the mass fraction of helium-4 should be larger than the standard theory predicts.

Method

Our first concern was the energy density of the WDM particles. We decided on the following distribution³:

$$\rho = \int f E d n = \frac{g}{(2\pi)^3} \int \frac{E d^3 p}{e^{(p-\mu)/T_*} + 1} = \frac{4\pi}{(2\pi)^3} \int \sqrt{m^2 + p^2} \frac{p^2 dp}{(e^{(p/T_*)} + 1)}$$

With $g=1$, $\mu=0$ and:

$$T_* = T_D \frac{R_D}{R}$$

Because both T_ν and T_* go as inverse R, we arrived at:

$$\frac{T_{*0}}{T_{\nu_0}} = \frac{T_*}{T_\nu}$$

And thus:

$$T_* = T_\nu \frac{T_{*0}}{T_{\nu_0}}$$

T_{*0} is found by the following sequence. Using $xT_* = p$ in the energy density we get:

$$\rho = T_*^3 \int \sqrt{m^2 + (xT_*)^2} \frac{x^2 dx}{e^x + 1}$$

At present, with $m \gg p$:

$$\rho = T_{*0}^3 m \int \frac{x^2 dx}{e^x + 1} = \frac{3}{2} T_{*0}^3 m \zeta(3)$$

As a fraction on the present day critical density:

$$\rho = \Omega_{WDM} \rho_C$$

From these, we arrive at an equation for T_{*0} :

$$T_{*0} = 2\pi \left(\frac{2}{12\pi} \frac{\Omega_{WDM} \rho_C}{m \zeta(3)} \right)^{1/3}$$

Using $\rho_C = 8.0992 h^2 \times 10^{-47} \text{ GeV}^4$, $h = 0.7$, and $\Omega_{WDM} = 0.3$:

$$T_{*0} = 2\pi \left(\frac{1}{6\pi} \frac{0.3 \cdot (0.7)^2 \cdot 8.0992 \times 10^{-47} \text{ GeV}^4}{\zeta(3)m} \right)^{1/3}$$

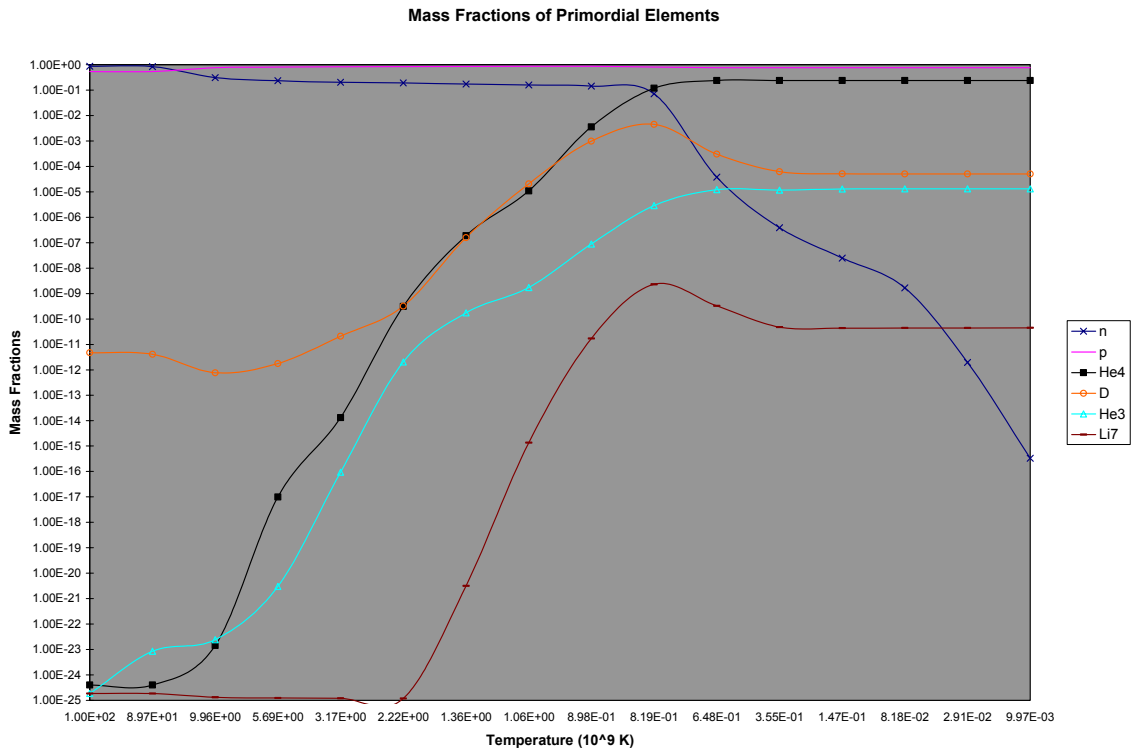
At the time of electron-positron annihilation, the neutrinos had already decoupled. As a result, the photon temperature was increased while the neutrino temperature was unaffected. We can calculate the present day value of the neutrino temperature as a

function of the photon temperature, which can be obtained from the microwave background radiation⁶:

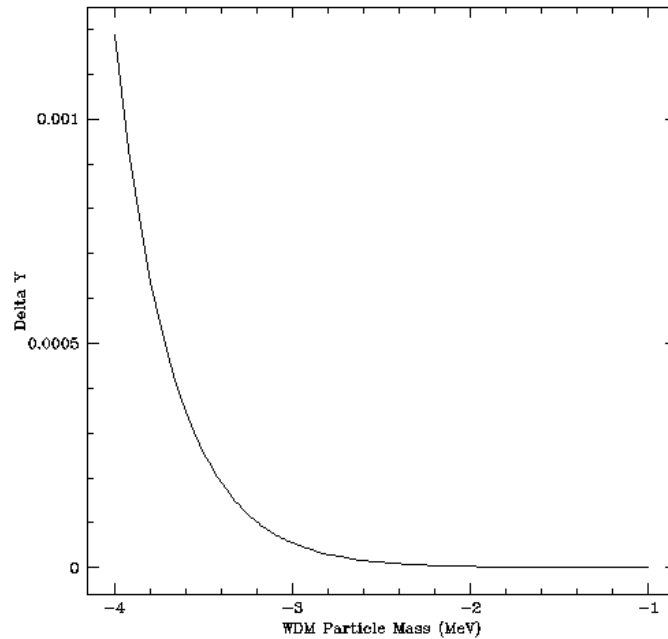
$$T_{\nu_0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma_0} = \left(\frac{4}{11}\right)^{1/3} (2.73K) = 1.95K$$

Having found the necessary equations for the energy density, we turned our attention towards modifying the Wagoner code. We internalized the equation for calculating T_* as a function of the WDM particle mass. After several attempts, the energy density equation was successfully modified to use the 5-point Gauss-Laguerre quadrature numerical integration method that is used in the code. The energy density for WDM particles could now be added to the preexisting energy density and the code run.

Results



First, using a baryon/photon ratio (η) of 4×10^{-10} , the helium-4 mass fraction (Y) was found for the case with no WDM being added. The value for Y computed was 0.241162. Subsequently, the code was run a number of times with WDM included before successful data emerged. The first good data that was obtained was for our three target masses of 1 keV, 10 keV, and 100 keV. For the $m=1$ keV case, the helium-4 mass fraction obtained was 0.241218 with $\Delta Y=0.000056$. For $m=10$ keV, we found $Y=0.241165$ with $\Delta Y=0.000003$. For $m=100$ keV, $Y=0.241162$ with $\Delta Y=0.000000$. Further data was obtained for mass values in the range from 0.1-100 keV. The results for ΔY are shown in the chart below:



Conclusions

As expected, the lower mass particles, possessing a higher thermal velocity, contributed more to the total energy density. This led to a greater increase in helium-4

abundance for the low mass WDM particles. For particles with mass about 1 keV or lower, the predicted ΔY would probably have been detectable by observations within the near future. However, for masses near 0.1 keV, the particle is very near the HDM limit, which has already been ruled out. Furthermore, recent simulations suggest that masses as high as 10 keV may be ruled out⁷. Given current experimental error limits, it seems unlikely that the ΔY values predicted for masses greater than 10 keV would be detectable in the foreseeable future⁸. Despite these facts, it is still hoped that this research has contributed to the body of knowledge on the subject of warm dark matter.

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⁶ Weinberg, S., 1972, *Gravitation and Cosmology*, (New York: John Wiley and Sons)

⁷ Yoshida, N., Sokasian, A., Hernquist, L., Springel, V., 2003, *Astrophys.J.* 591, L1-L4

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