Quasiparticle Description of the Quark Gluon Plasma

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Abstract

We create a quasiparticle equation of state of the quark gluon plasma which builds on a phenomenological model by parameterizing the onset of confinement near the phase transition. A model for the thermal mass is used which incorporates the dropping of the mass near the critical temperature for the phase change. The results are compared with existing data to ensure the accuracy of the new calculation. Lattice results of pressure, energy density and entropy density are reproduced by the confinement model with excellent accuracy for the pure gluonic plasma. The accuracy of the model is explored with dynamical quarks and converted to a form $p = p(\varepsilon)$ that can be used in hydrodynamic simulations.

1 Introduction

Lattice QCD simulations suggest that there exists a phase transition (or crossover) from a confined hadronic phase to a deconfined state known as the quark gluon Plasma (QGP), where quarks and gluons define the relative degrees of freedom, at a critical temperature of $T_c \sim 150 - 170$ MeV [1]. Understanding the QGP is of particular importance for describing the evolution of the universe after the Big Bang. The QGP can be studied using ultrarelativistic heavy-ion collisions at the CERN SPS and at RHIC. Upon the collision of two nuclei (often Pb+Pb or Au+Au collisions), a "fireball" develops in the impact region. This hot matter collectively expands outward due to a large pressure gradient within the fireball and a vacuum outside of the plasma. As the fireball expands, it cools and eventually reaches T_c, where the phase change to a hadronic gas occurs. The rate of expansion is controlled by the interplay between the pressure, which pushes the matter outward, and the energy density, which provides inertia and restricts the expansion. In order to properly study the dynamics of the QGP, a statistical parametrization of pressure p, energy density ε and entropy density s must be defined to create the equation of state (EOS). The derivation of the EOS for the QGP from first principles becomes increasingly difficult when approaching T_c because perturbative calculations are not expected to be applicable near the phase transition [2]. Thus numerical simulations of lattice QCD needed to extract the QGP EOS to high accuracy. What we will do is to create an EOS from thermodynamic principles which emulates the lattice QCD data.

Below T_c, matter exists as a hadron resonance gas. This describes both stable (on strong interaction time scales) hadrons and resonance states of these particles. Only a limited number of hadronic states with approximate masses ≤ 1.5GeV are needed in this calculation, due to the suppression of higher mass states [3]. Though it does not by itself exhibit a phase change, a purely hadronic EOS will also be derived and used to simulate matter below T_c.

Above T_c, most of the usually employed descriptions of the QGP describe an ideal gas of noninteracting quarks and gluons. This model has great difficulty reproducing lattice data and can be used only as a rough approximation. A phenomenological description treats the interacting massless quarks and gluons as noninteracting, massive quasiparticles [4]. The quasiparticle picture treats these quasiparticle degrees of freedom in a similar way as one treats electrons in condensed matter theory: the interactions with the medium gives them a dynamical mass, and by this effect most of the interactions can be efficiently taken into account. While this model does describe the lattice data close to T_c, it has trouble explaining the observed dropping of the thermal gluon screening mass near the phase transition [3]. Therefore a different phenomenological quasiparticle model will be used as the framework for an EOS that no longer treats the quasiparticles as completely noninteracting. Rather a parametrization of the confinement of particles will be used to better simulate the QGP near the phase transition.

2 Hadron Resonance Gas

A realistic thermodynamic parametrization should describe a smooth but rapid transition from hadronic matter to the QGP, characterized by a large and rapid change of entropy density near T_c [3]. If one matches approximate equations of state below and above T_c by a thermodynamic Maxwell construction at T_c, one obtains instead a first order phase transition with a large discontinuity for ε and s at T_c. To obtain a smooth transition without large discontinuities, such as the one obtained by lattice QCD, considerable refinement of the quasiparticle model is necessary.

The calculations for the hadron gas can be restricted to the following particles and their antiparticles [3]:

$$\pi, K, \eta, \rho, \omega, K^*, p, n, \eta', \phi, \Lambda, \Sigma, \Delta, a_1, \Xi, \Sigma(1385). \quad (1)$$

If we assume a phase transition at $T_c \approx 170$ MeV, the resonance states of hadrons with higher mass will be sufficiently suppressed and are negligible. With the appropriate particles defined, one can begin the discussion with the grand canonical partition function for a noninteracting resonance gas:

$$\mathcal{Z}^{\mathrm{H}}(T, V, \mu_{\mathrm{B}}, \mu_{\mathrm{S}}) = \prod_{\mathrm{h}} \exp[\mathrm{Z_h}(T, V, \mu_{\mathrm{h}})], \quad (2)$$

Here V is the fireball volume, and the product is over all the hadron species. The chemical potential μ_{h} of the hadron h is related to its baryon number $\mathrm{B_h}$ and strangeness $\mathrm{S_h}$ through

$$\mu_{\mathrm{h}} = \mathrm{B_h}\mu_{\mathrm{B}} + \mathrm{S_h}\mu_{\mathrm{S}}. \quad (3)$$

This ensures conservation of net baryon number and strangeness as required by the conservation laws for strong interactions. At high energies, such as those provided by RHIC, the collision fireball is effectively particle - antiparticle symmetric. This implies $\mu_{\mathrm{B}} = 0$ and $\mu_{\mathrm{S}} = 0$. We can therefore let $\mu_{\mathrm{h}} = 0$ for all hadrons. With vanishing chemical potential, both net baryon density and net strangeness density will be zero, and the EOS will be greatly simplified.

The partition function for each hadron species $Z_{\mathrm{h}}(T, V, \mu_{\mathrm{h}} = 0)$ will be used to define pressure, entropy density, and energy density. For a noninteracting gas of hadrons with mass m_{h}, it is given by

$$Z_{\mathrm{h}}(T, V) = \frac{V}{T} p_{\mathrm{h}} = \frac{\mathrm{g_h} V}{6\pi^2 T} \int_{m_{\mathrm{h}}}^{\infty} dE \frac{(E^2 - m_{\mathrm{h}}^2)^{\frac{3}{2}}}{e^{E/T} \pm 1}, \quad (4)$$

where g_{h} is the particles spin (S)-isospin (I) degeneracy factor defined as $g_{\mathrm{h}} = (2S+1)(2I+1)$, and m_{h} is the mass. The partition function contains the Fermi-Dirac or Bose-Einstein distribution function, corresponding to the ± 1

in the denominator for fermions and bosons respectively. From the partition function of each species, one can derive all thermodynamic quantities from basic thermodynamic principles [3, 5]:

$$p(T) = T\frac{\partial \ln \mathcal{Z}^{\mathrm{H}}}{\partial V} = \frac{T}{V}\ln \mathcal{Z}^{\mathrm{H}} = \sum_{\mathrm{h}} \frac{g_{\mathrm{h}}}{6\pi^2}\int_{m_{\mathrm{h}}}^{\infty} dE \frac{(E^2 - m_{\mathrm{h}}^2)^{\frac{3}{2}}}{e^{E/T} \pm 1}, \tag{5}$$

$$\varepsilon(T) = \frac{-1}{V}\frac{\partial \ln \mathcal{Z}^{\mathrm{H}}}{\partial \frac{1}{T}} = \sum_{\mathrm{h}} \frac{g_{\mathrm{h}}}{2\pi^2}\int_{m_{\mathrm{h}}}^{\infty} dE \frac{E(E^2 - m_{\mathrm{h}}^2)^{\frac{1}{2}}}{e^{E/T} \pm 1}, \tag{6}$$

$$s(T) = -\frac{\partial p(T)}{\partial T} = \frac{p(T)}{T} + \sum_{\mathrm{h}} \frac{g_{\mathrm{h}}}{2\pi^2}\int_{m_{\mathrm{h}}}^{\infty} dE \frac{E^2}{T}\frac{(E^2 - m_{\mathrm{h}}^2)^{\frac{1}{2}}}{e^{E/T} \pm 1}, \tag{7}$$

$$\rho_{\mathrm{B}}(T) = \frac{T}{V}\frac{\partial \ln \mathcal{Z}^{\mathrm{H}}}{\partial \mu_{\mathrm{B}}} = 0, \tag{8}$$

$$\rho_{\mathrm{S}}(T) = \frac{T}{V}\frac{\partial \ln \mathcal{Z}^{\mathrm{H}}}{\partial \mu_{\mathrm{S}}} = 0, \tag{9}$$

where $p(T)$, $\varepsilon(T)$ and $s(T)$ are related by the thermodynamic principle $sT = \varepsilon + p$, and ρ_{B} and ρ_{S} are the net baryon density and net strangeness density respectively. Pressure can then be solved in terms of energy density ε and net baryon density ρ_{B} to obtain a form $p(\varepsilon, \rho_{\mathrm{B}})$ which can be used for hydrodynamic simulations. However, with the zero chemical potential there will be a vanishing net baryon density and one will have simply $p = p(\varepsilon)$. Once in this form, the EOS $p(\varepsilon) = \frac{\varepsilon}{3}$ is valid for an ideal gas of noninteracting particles, and can be used as the asymptotic limit to the realistic EOS for $T \to \infty$.

3 Pure Gluonic Plasma

3.1 Quasiparticle Model

Above T_c the quarks and gluons no longer interact as strongly as in the hadron gas where they bind to color neutral objects called hadrons. Instead they are essentially "deconfined". Once the phase change occurs, there are still residual interactions among the gluons and quarks, forming the quasiparticles. This model is based on the idea that in strongly interacting systems, often much of the physics can be described by effective degrees of freedom [2]. We will begin by examining a plasma of pure glue with three colors

$N_C = 3$. Again, the EOS will assume a vanishing chemical potential in order to discard the complication of particle-antiparticle inequality.

Expecting that gluons maintain their effective number of degrees of freedom, the pressure can be derived from the principle

$$p(T) = \nu_g \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{3E_k} \mathcal{F}_{\rm B}(E_k). \tag{10}$$

Here, $\nu_{\rm g} = 2(N_C^2 - 1)$ is the degeneracy factor, k is the momentum, $E_k = \sqrt{k^2 + m_g^2(T)}$ where $m_g(T)$ is the gluon's quasiparticle mass in the medium, and

$$\mathcal{F}_{\rm B}(E_k) = \frac{1}{e^{E_k/T} - 1}, \tag{11}$$

is the thermal Bose-Einstein distribution function. We find

$$p(T) = \frac{\nu_g}{6\pi^2} \int_{m_g(T)}^{\infty} dE \frac{(E^2 - m_g^2(T))^{\frac{3}{2}}}{e^{E/T} - 1} - B(T). \tag{12}$$

The pressure now involves a temperature dependent function $B(T)$ in order to satisfy the relation $s(T) = \frac{\partial p(T)}{\partial T}$. The energy density ε and entropy density s can also be derived,

$$\varepsilon(T) = \frac{\nu_g}{2\pi^2} \int_{m_g(T)}^{\infty} dE \frac{E^2(E^2 - m_g^2(T))^{\frac{1}{2}}}{e^{E/T} - 1} + B(T), \tag{13}$$

$$s(T) = \frac{\nu_g}{2\pi^2 T} \int_{m_g(T)}^{\infty} dE \frac{\frac{1}{3}(4E^2 - m_g^2(T))}{E} \frac{(E^2 - m_g^2(T))}{e^{E/T} - 1}. \tag{14}$$

Note that the entropy has a simple quasiparticle form whereas ε and p do not. They involve an extra function $B(T)$ which is called the "bag constant", which incorporates all extra functions created by the relation $s(T) = \frac{\partial p(T)}{\partial T}$. It is used to maintain thermodynamic consistency, and must have opposite signs in p and ε to satisfy $Ts = \varepsilon + p$. In Eq. (13) the bag constant adds to the energy density and can thus be interpreted as the thermal vacuum energy density [2].

A crucial part to the quasiparticle model is the temperature dependent mass $m_g(T)$. It absorbs the main effects of the interactions among the gluons in the dense QGP giving them a dynamical mass. Since the density of the QGP depends on T, so does the dynamical mass $m_g(T)$.

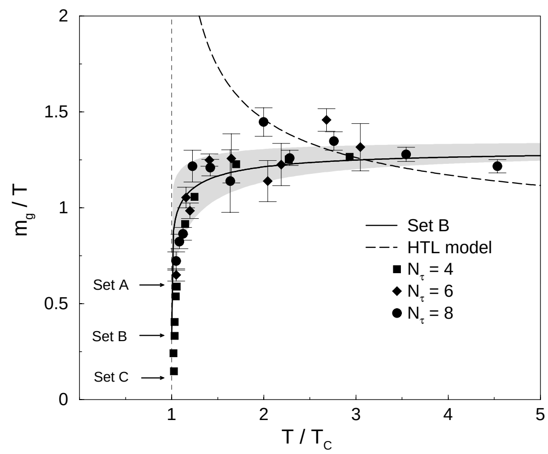

Figure 1: A sketch of the temperature dependent gluon mass $m_g(T)$ in a parametrization B of the confinement model [2] and the phenomenological quasiparticle model [4]. Near T_c, lattice data fit well with the drop in the thermal mass of the confinement model [2]

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While the phenomenological quasiparticle model does take into account dynamical mass generation, it models the quasiparticles masses in such a way that the mass begins to increase rapidly as T_c is approached from above. The phenomenological mass is defined in the basic form $m_g(T) = G(T)T$ where $G(T)$ describes the effective coupling of the particles. While the increase in mass near the phase transition does not agree with lattice data, the basic form of the mass will be kept. We will use for an approximate coupling constant

$$G(T) = G_0\left([1+\delta] - \frac{T_c}{T}\right)^{\beta}, \tag{15}$$

with $G_0 = 1.3$, $\delta = 10^{-6}$, and $\beta = 0.1$ for the case of pure gluons. These parameters are extracted from a fit to the lattice QCD data as described in table 1 of Ref. [2]. The temperature dependent mass becomes

$$m_g(T) = G_0 T\left([1+\delta] - \frac{T_c}{T}\right)^{\beta}. \tag{16}$$

As seen in figure 1, the effective gluon mass drops near the phase change, and agrees well with QCD data.

	G_0	δ	β	C_0	δ_c	β_c
Set A	1.35	10^{-5}	0.2	1.24	0.0029	0.34
Set B	1.30	10^{-6}	0.1	1.25	0.0026	0.31
Set C	1.30	10^{-7}	0.05	1.27	0.0021	0.30

Table 1: Parametrizations for the coupling function $G(T)$ and the corresponding confinement factor $C(T)$ [2].

3.2 Confining the Quasiparticles

While the quasiparticle model gives an accurate approximation to the QGP EOS, it begins to break down near T_c because it neglects the process of (de)confinement. As the phase transition is approached from below T_c, the gluons gain enough energy to be freed, which causes a large increase in energy and entropy density. Since the energy density, pressure, etc. drop when approaching T_c from above, it appears that the gluons lose their relative degrees of freedom and the process of confinement takes place as the gluons become trapped in groups of gluons called glueballs [2]. Essentially, confinement is a process which decreases the number ν_g of thermally active degrees of freedom within the QGP. The degeneracy factor can be revised to include a temperature dependent confinement factor $\nu_g \rightarrow C(T)\nu_g$. With the confinement factor, the following equations are obtained:

$$p(T) = \frac{C(T)\nu_g}{6\pi^2} \int_{m_g(T)}^{\infty} dE \frac{(E^2 - m_g^2(T))^{\frac{3}{2}}}{e^{E/T} - 1} - B(T), \tag{17}$$

$$\varepsilon(T) = \frac{C(T)\nu_g}{2\pi^2} \int_{m_g(T)}^{\infty} dE \frac{E^2(E^2 - m_g^2(T))^{\frac{1}{2}}}{e^{E/T} - 1} + B(T), \tag{18}$$

$$s(T) = \frac{C(T)\nu_g}{2\pi^2 T} \int_{m_g(T)}^{\infty} dE \frac{\frac{1}{3}(4E^2 - m_g^2(T))}{E} \frac{(E^2 - m_g^2(T))}{e^{E/T} - 1}. \tag{19}$$

At temperatures much greater than the critical temperature $T \gg T_c$, the confinement factor must play little, if no, role in the EOS. As the phase transition is approached from above, confinement must set in, and the confinement factor must act to limit the thermally active degrees of freedom. An appropriate $C(T)$ to fit such an effect will take the form

$$C(T) = C_0([1 + \delta_c] - \frac{T_c}{T})^{\beta_c}, \tag{20}$$

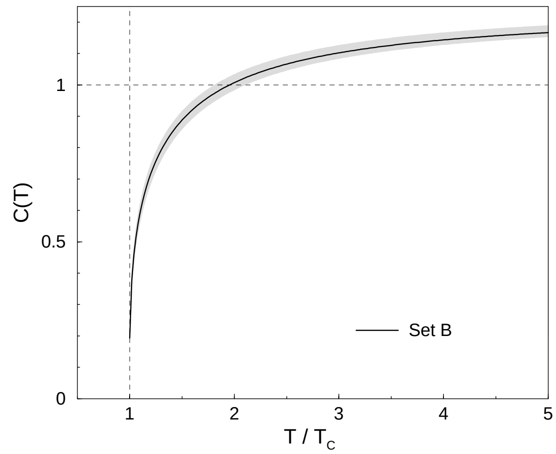


Figure 2: The confinement factor $C(T)$ as a function of temperature. The gray band shows the range for the corresponding mass parametrizations of table 1. The solid line is obtained from set B [2].

where $C_0 = 1.25$, $\delta_c = .0026$, and $\beta_c = 0.31$ for the case of pure gluonic matter are the parameters extracted from a fit of lattice QCD data [2]. In figure 2, the plot of $C(T)$ displays the process of confining the active degrees of freedom. The confinement is seen to be very weakly mass dependent, the difference from set A to set C is small. Set B will be used for further analysis of pure gluonic matter.

The bag constant $B(T)$ is strongly dependent on this confinement process and the thermally dependent masses because it is formed from the extra variables of the temperature derivative of the pressure. It will take the form [2]

$$B(T) = B_1(T) + B_2(T) + B_0, \tag{21}$$

where B_0 is a constant used to match the hadronic EOS to the QGP and

$$B_1(T) = \frac{\nu_g}{6\pi^2}\int_{T_c}^{T} d\tau \frac{dC(\tau)}{d\tau}\int_0^\infty dk \mathcal{F}_{\mathrm{B}}(E_k)\frac{k^4}{E_k}, \tag{22}$$

$$B_2(T) = -\frac{\nu_g}{4\pi^2}\int_{T_c}^{T} d\tau C(T)\frac{dm_g^2(\tau)}{d\tau}\int_0^\infty dk \mathcal{F}_{\mathrm{B}}(E_k)\frac{k^2}{E_k}. \tag{23}$$

It is easy to see that the terms $\frac{\partial B(T)}{\partial T}$ cancel the unwanted terms in the entropy density $s = -\frac{\partial p}{\partial T}$ which result from the temperature dependences of

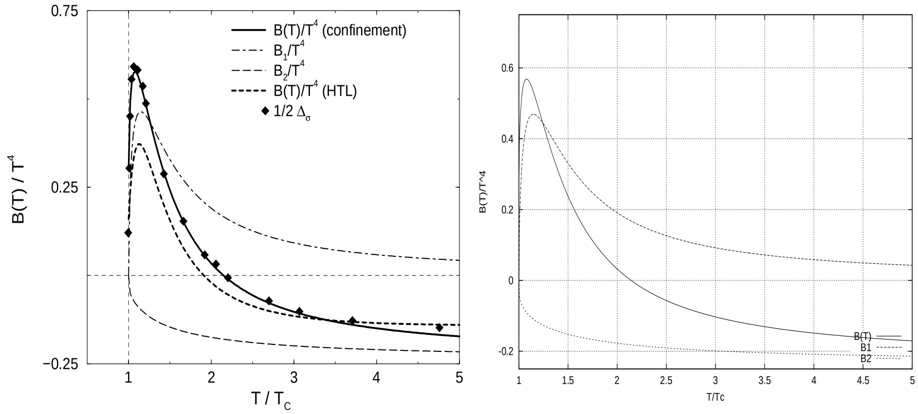


Figure 3: The background field $B(T)$ and its components B_1 and B_2, defined in Eq. (21). Also shown is $B(T)$ in the HTL model. Symbols display the spacelike plaquette expectation value $\frac{1}{2}\Delta_\sigma$ taken from the lattice calculation of Ref. [7].

$C(T)$ and $m_g(T)$. The integration constant B_0 was set to $0.3T^4$, and the gluon degeneracy fact $\nu_g = 2(N_C^2 - 1) = 16$. In figure 3, one can see the shape of the function $B(T)$, and its components B_1 and B_2. Figure 3 (right) is a numerical calculation done using the confinement data. Comparing with the left figure, little if no difference can be seen. This second calculation was done to reproduce the data of Schneider and Weise in Ref. [2]. In this model, B_2 maintains a negative and decreasing value for all temperatures $T \geq T_c$. This is a unique feature to the confinement model in that the function B_1 is solely responsible for creating the peak of $B(T)$ [2]. The bag function of the confinement model reaches a peak value approximately 1.6 times larger and goes through zero at a temperature approximately $0.2T_c$ greater than the quasiparticle model of Ref. [4]. It is interesting to note that the shape of $B(T)$ now closely resembles the interaction measure $\Delta = \frac{(\varepsilon - 3p)}{T^4}$ extracted from the lattice data where one finds $B(T) \approx \frac{1}{2}\Delta_\sigma(T)$ [2].

Figure 4 presents the results of the confinement model to lattice data. The left figure is the calculation from Ref. [2] while the figure on the right displays my own numerical calculations. The agreement serves as a check for my program to ensure that I am using the correct algorithm to compute the EOS. Both figures agree with lattice data near T_c but seem to deviate slightly $T \approx 4T_c$ dues to the imperfect parametrization of $C(T)$ [2].

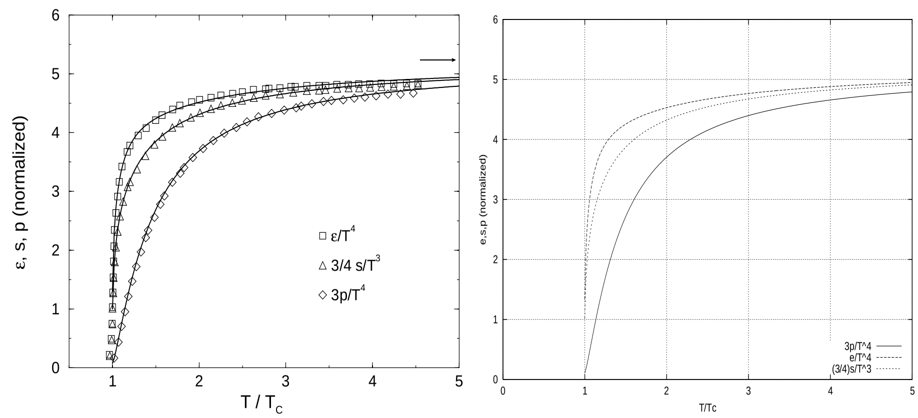


Figure 4: Comparisons of normalized pressure, energy density and entropy density with the lattice data. The right figure is a recent numerical calculation of the confinement data for a pure gluonic plasma.

4 Quasiparticle Model with Quarks

4.1 Temperature Dependent Bare Masses

Unfortunately, the addition of dynamical quarks into the EOS is not simple. No extrapolation of the QCD EOS with realistic quark masses exists to date [2]. However, one can extend the confinement model to construct an EOS for dynamical masses in order to emulate lattice QCD data.

For temperatures of the order of $T_c \approx 170$ MeV, only the up (u), down (d), and strange (s) quarks are light enough to be deconfined. To account for temperature dependent bare masses, two light flavored quarks with $\frac{m_{q,0}}{T} = 0.4$ and one heavy quark, the strange quark, with $\frac{m_{s,0}}{T} = 1.0$, will be used in calculations [2] where the bare mass $m_{q,0}$ can be interpreted as the rest mass of the particle. We use a number of flavors $N_{\mathrm{f}} = 2, 2+1, 3$, where 2+1 flavors takes into account the difference in bare mass of the strange quark and will set $N_{\mathrm{f}} = 2.3$, while 3 flavors will treat all quarks as light with light bare masses.

With the addition of dynamical quarks, the expressions for the thermal masses must be revised. The masses of both the gluons and quarks must take into account the number of flavors N_{f} and the number of colors N_{C}. For

	C_0	δ_c	β_c
2 flavors	1.25	0.02	0.28
2+1 flavors	1.16	0.02	0.29
3 flavors	1.03	0.02	0.2
gluon	1.25	0.0026	0.31

Table 2: Parametrizations of equation (20) for the confinement function $C(T)$ in the presence of dynamical quark flavors. For comparison, the corresponding values of the pure gauge system (set B) are also shown [2].

the thermal gluon mass, we use:

$$\frac{m_g(T)}{T} = \sqrt{\frac{N_\mathrm{C}}{6} + \frac{N_\mathrm{f}}{12}}\tilde{\mathrm{g}}(T, N_\mathrm{C}, N_\mathrm{f}) \tag{24}$$

with the expression for the effective coupling [2]:

$$\tilde{\mathrm{g}}(T, N_\mathrm{C}, N_\mathrm{f}) = \frac{\mathrm{g}_0}{\sqrt{11N_\mathrm{C} - 2N_\mathrm{f}}}\left([1+\delta] - \frac{T_\mathrm{C}}{T}\right)^{\beta}, \tag{25}$$

where $\mathrm{g}_0 = 9.4$, $\delta = 10^{-6}$, and $\beta = 0.1$ are the parameters extracted from the lattice QCD data and will be taken to be universal [2]. The thermal quark mass will be modeled as:

$$\frac{m_q(T)}{T} = \sqrt{\left(\frac{m_{q,0}}{T} + \sqrt{\frac{N_\mathrm{C}^2 - 1}{16N_\mathrm{C}}}\tilde{\mathrm{g}}(T)\right)^2 + \frac{N_\mathrm{C}^2 - 1}{16N_\mathrm{C}}\tilde{\mathrm{g}}(T)^2}. \tag{26}$$

It is now straightfoward to extend Eqs. (17)-(19) and the bag constant $B(T)$ to include dynamical quarks. The pressure is expressed as:

$$\begin{aligned} p(T) = \frac{C(T)\nu_\mathrm{g}}{6\pi^2}\int_{m_g(T)}^{\infty} dE\frac{((E^\mathrm{g})^2 - m_g^2(T))^{\frac{3}{2}}}{e^{\frac{E^\mathrm{g}}{T}} - 1} \\ + \sum_{i=1}^{N_\mathrm{f}} \frac{C(T)2N_\mathrm{C}}{3\pi^2}\int_{m_i(T)}^{\infty} dE\frac{((E^i)^2 - m_i^2(T))^{\frac{3}{2}}}{e^{\frac{E^i}{T}} - 1} - B(T). \end{aligned} \tag{27}$$

Again, $E_k^\mathrm{g} = \sqrt{k^2 + m_\mathrm{g}^2(T)}$ is the energy-momentum relation for the gluons and $E_k^i = \sqrt{k^2 + m_i^2(T)}$ for each quark flavor $q = i$, with bare quarks masses $m_{q,0}$ [2]. The bag function $B(T)$, entropy density s, and the energy density ε all follow analogously. In order to maintain agreement with the lattice data, the parameters C_0, δ_C and β_C of the confinement model must be slightly modified. The flavor dependence of the confinement $C(T)$, as extrapolated from [2], can be seen in table 2.

In figure 5, the pressure is plotted for 2 and 2+1, $N_\text{f} = 2.3$, flavors. The parameters of $C(T)$ can be found in table 2. The factor B_0 is set to $1.4T_c^4$ [2]. My numerical calculation was performed to recreate the data in [2]. As one can see, the confinement model matches closely with the lattice QCD calculations, especially for temperatures $T \leq 3T_c$.

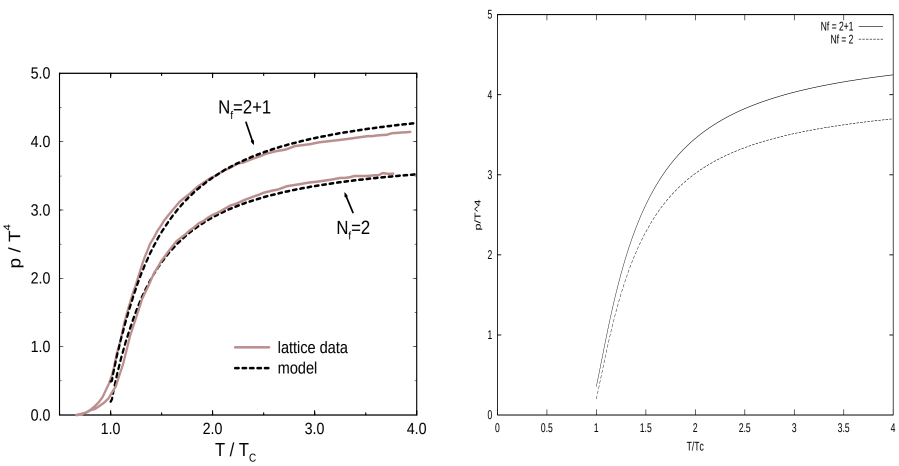


Figure 5: Left: The rescaled lattice pressure $p_{cont} \approx 1.1 p_{lat}$ (gray lines) compared to the confinement quasiparticle model with temperature dependent bare masses. Right: A recreation of the confinement quasiparticle model using parametrizations of [2].

4.2 Realistic Quark Masses

In this section, one will concentrate on fitting the quasiparticle confinement model to QCD results using physical masses. In Eq. (26), $m_{q,0}$ will be set to the real-world values of $m_{u,d} \approx 0$ MeV and $m_s \approx 170$ MeV. This model assumes that $C(T)$ does not depend on $m_{q,0}$ which is not obviously correct [2]. However, even with this assumption, the confinement data was able produce data that agreed with lattice QCD results. Figure 6 shows the results for pressure, energy density and entropy density using $N_\mathrm{f} = 2$. My calculations were done to reproduce and check the data produced by the confinement model. One can see the confinement model lies within the estimate of the continuum EOS for massless two flavor QCD results.

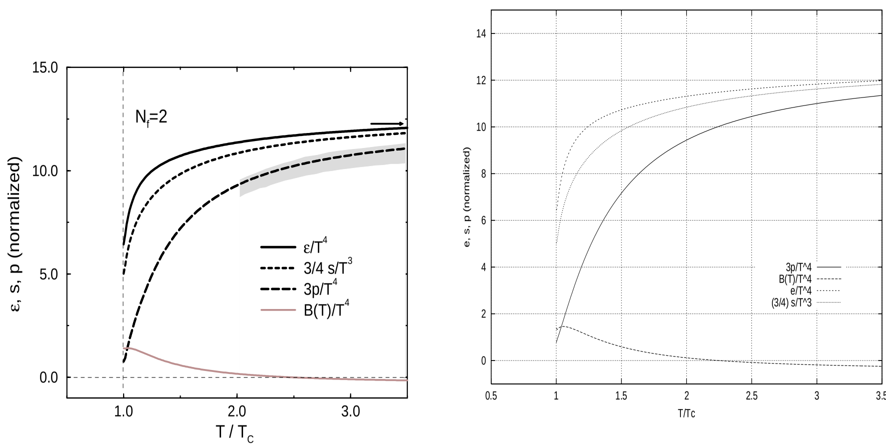
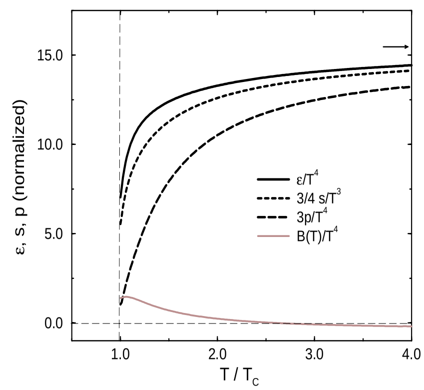


Figure 6: Pressure, energy density, entropy density, and bag function of the confinement model using two light quark flavors. The arrow indicates the ideal gas limit and the shaded region of the left figure indicates an extrapolation of the QCD data range [2].

For the case of three quark flavors, $N_\mathrm{f} = 2.3$, numerical calculations of the confinement model did not reproduce the same data found in [2]. At T_c, the pressure, energy density and entropy density were found to have slightly higher values than the data found in [2] and maintained higher values through $T \approx 4T_c$. Unfortunately, the source of this discrepancy is unknown. However it can be seen that in both figures the results approach the ideal gas limit slower than the two flavor case due to the mass suppression of the third heavier strange quark [2].

4.3 Fitting Data to Hydrodynamics

Hydrodynamic simulations can be done to study the development of the QGP. Differences in fireball expansion due to radial or elliptic flow can be studied and explored. In order to proceed with these simulations, one must have an equation of state of the form $p = p(\varepsilon, \rho_\mathrm{B})$. Since I am working under



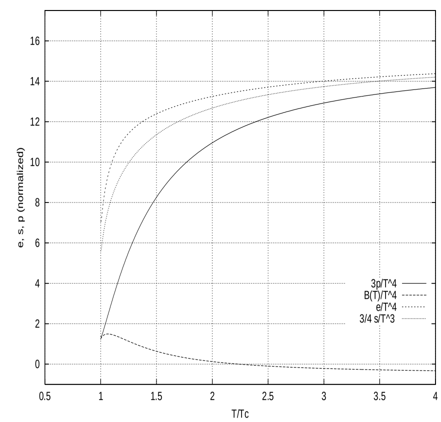


Figure 7: Pressure, energy density, entropy density, and the bag function for 2+1 flavor confinement data. In order to produce these curves we take $N_{\mathrm{f}} = 2.3$. Notice the slight elevation of the data in my curves of the right, as opposed to the curves on the left found in [2]

the condition $\mu = 0$, the previous data of the form $p = p(T)$ was converted to the form $p = p(\varepsilon)$.

It is now easy to view the EOS of the QGP, and see the phase transition of the energy dependent pressure. The resulting EOS has a smooth phase transition, and no longer resembles the cornered EOS used previously. It should also be noted that as one moves farther away from the phase transition, the EOS approaches the ideal gas limit of $p = \frac{1}{3}\varepsilon$.

5 Summary

A thermodynamic description of the QGP has now been presented using a quasiparticle model of the EOS. This model differs from the previously used phenomenological model [4], with the inclusion of a confinement factor, which is used to reduce the number of thermally active degrees of freedom of the system, and a thermal mass that is seen to drop as the critical temperature is approached from above. The inclusion of these factors create extra terms within the thermodynamic expressions of the EOS and lead to the inclusion of a bag function $B(T)$ used to hold all extra terms.

Pressure, energy density, and entropy density were numerically calculated

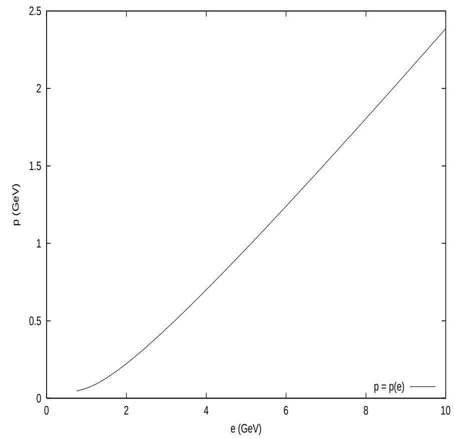


Figure 8: Pressure plotted as a function of energy density. The ideal gas limit of $p = \frac{1}{3}\varepsilon$ is approached at large values of ε. This form of data can be used in hydrodynamic simulations.

using this model and compared to the existing data for the confinement model in Ref. [2] and to lattice QCD data. In the case of pure gluonic matter, the data from Re. [2] was reproduced. This model provided an excellent description of lattice data. Similarly, including temperature dependent bare masses reproduced existing confinement data, however the inclusion of dynamical masses is yet to reproduce QCD data extremely well.

For systems with physical quark masses, a discrepancy was found between my EOS and that of Ref. [2]. The source of this discrepancy is not yet known, but will be explored in future research. This data was then transformed into a useful form for hydrodynamic simulations. Using this quasiparticle model with physical quark masses still does not reproduce QCD data accurately enough. Further simulations of the confinement model and of lattice QCD are needed to validate the accuracy of the confinement model.

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