

*The Effect of the Primordial Helium-4  
Abundance on the Power Spectrum of the  
Cosmic Microwave Background Radiation*

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## ***ABSTRACT***

The effect of the amount of helium-4 produced from Big Bang nucleosynthesis on the power spectrum of the cosmic microwave background radiation (CMBR) is explored for two reasons: to see if the mass fraction of helium-4 produced during the events following the Big Bang has an effect on the power spectrum of the CMBR, and if a constraint can be placed on this amount. This is done through the help of a computer program called CMBFAST, which computes power spectra of models of the CMBR with varying parameters. The effect was indeed apparent in graphs produced from values taken from CMBFAST. Drawing randomly on numbers from a Gaussian distribution of the parameters that were taken to vary in the program, bands of power spectra for a specific value of the mass fraction of helium-4 were produced, and a constraint could be placed on the amount of helium-4 present in the early universe. An upper limit of eighty percent helium-4 by mass was confidently found to exist through computer modeling, while a lower limit was inconclusive.

### ***1. Introduction***

Mysteries abound in the study of the universe. Cosmologists struggle with the formation of large-scale structure, the existence of dark matter, the Hubble constant, and the evidence for the existence of the cosmological constant, along with many other questions that have yet to be answered. The discovery of the cosmic microwave background radiation brought hope that some of the questions might soon be answered, and astrophysicists and cosmologists have been analyzing the data that have come back from studies of this radiation, parading under the names of COBE, BOOMERANG, MAXIMA, and DAS1, as well as MAP, which only recently was launched. These studies have lent credence to a flat universe (as opposed to closed or open), as

well as provided hope for narrowing down the ranges of values for such things as the baryon density of the universe and the dark matter density, along with many others.

## ***2. Background Information***

### ***2.1 Primordial Nucleosynthesis***

#### ***2.1.1 Element Production***

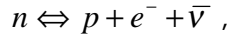
The Big Bang possessed an immense amount of energy, an amount only dreamed of by physicists with particle accelerators. The Big Bang itself can be thought of as the pinnacle of particle accelerators. At the beginning, the energy was so high that, theoretically, all the forces of the universe were united. As time went on, the universe began to cool, the forces began to become distinct, and particles began to form. Soon, the universe was a sea of protons, neutrons, and leptons, which include electrons and neutrinos. These particles had very specific interactions. These consist of

$$\bar{\nu} + p \Leftrightarrow e^+ + n$$

and

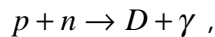
$$\nu + p \Leftrightarrow e^- + n ,$$

where  $\nu$  is a neutrino,  $\bar{\nu}$  is an antineutrino,  $p$  is a proton,  $n$  is a neutron,  $e^-$  is an electron, and  $e^+$  is a positron. Since the neutron is slightly more massive than the proton, the universe favored the reaction to form a proton more than that to form a neutron. This led to an imbalance in the number of neutrons and protons, which had been essentially equal at the beginning of this era of the universe (Hawley and Holcomb 362-368). There is also another equation that contributed to the imbalance of neutrons and protons, namely



which is the decay reaction for neutrons, as a free neutron is not stable (Olive, et. al. 3).

The universe continued to lose energy through expansion, thereby cooling, and soon neutrinos no longer interacted with matter, and the energy of the universe dropped enough that electrons could not be created, leaving the ratio of neutrons to protons  $n/p$  fixed with respect to the first two reactions (Hawley and Holcomb 368). This was also known as “freeze-out” of the ratio of the nucleons, and  $n/p$  continued to drop due to free neutron decay. Next came the step in the universe known as nucleosynthesis, or the creation of elements due to nuclear reactions. The elements that were created during this time in the history of the universe were deuterium (D), helium-3 ( $^3\text{He}$ ), helium-4 ( $^4\text{He}$ ), and lithium-7 ( $^7\text{Li}$ ) (hydrogen was of course already in existence, since it consists of one proton). The first step in the nucleosynthesis chain dealt with deuterium,



where  $\gamma$  is a photon. Soon after this reaction became stable, most deuterium formed by it fused with another proton to form  $^3\text{He}$ , and most of those combined with another neutron to form  $^4\text{He}$ . By knowing the approximate ratio of neutrons to protons, one can estimate the mass fraction of the universe that was composed of  $^4\text{He}$  from these reactions. By this time, there were approximately seven protons for every neutron, and by using the equation below, the value can be estimated:

$$Y_p \approx \frac{2(n/p)}{1 + (n/p)} = 0.25 ,$$

where  $Y_p$  is another way of representing the primordial abundance of  $^4\text{He}$  (Olive, et. al. 4). This is a direct result of knowing the baryon density of the universe, where a baryon is another name for a proton or a neutron. If the ratio of neutrons to protons were one at the beginning of

nucleosynthesis, then almost all, if not all, of the matter in the universe would have been in the form of  ${}^4\text{He}$ . The amount of deuterium is a good indicator of the baryon density, as it is easily fused with other particles to get more complex nuclei, and its abundance decreases with increasing baryon density. A small amount of  ${}^7\text{Li}$  is produced through fusion of this sort, and is also a good indicator of the baryon density, as it is destroyed with increasing amounts of particles (Olive, et. al. 4).

### ***2.1.2 Recombination***

Through this analysis of the production of elements in the early universe, one can see how a change in the amount of one type of element that was produced could have large ramifications for later periods of the lifetime of the universe. This is very true with respect to the time of the period of recombination, or decoupling, when matter separated from radiation. As the universe aged, it continued to go through different epochs as it cooled, where certain types of particles dominated. Eventually, the radiation era was reached, where radiation was the dominant form of energy in the universe, existing in the form of photons. These photons interacted with the matter that was around them, and the temperature was such that the photons kept all the elements that were formed during nucleosynthesis ionized. “During this epoch, the electrons glue the baryons to the photons by Compton scattering and electromagnetic interactions. The dynamics that result involve a single photon-baryon fluid” (Hu, 3). Compton scattering is basically a high-energy photon (either x-ray or gamma ray) colliding with an electron and “scattering” off of it, thereby ionizing an atom with a low binding energy, such as those that existed in the early universe (Halliday, et. al. 990). However, the universe continued to cool as it expanded, and soon the photons were not energetic enough to keep the electrons away from the nuclei, and neutral atoms formed. This is when matter and radiation separated

(hence the name decoupling), and it heralded the next stage in the history of the universe, the matter-dominated era, which is its present condition (Hawley and Holcomb 376-377). These photons that were then free to wander about the universe continued to redshift as the universe expanded, forming the cosmic microwave background, as described below. The picture of the last time that matter and radiation interacted is called the surface of last scattering, and the size of this surface (the size of the expanded universe when photons separated from matter) depends on the time of decoupling, which depends on the ratio of neutrons to protons, which is related to the baryon density, and is manifested in the amount of  $^4\text{He}$  produced in the early universe.

Since the universe was, according to the standard theory, approximately seventy-five percent hydrogen at the time of decoupling, the photons had to decrease in temperature to something below the binding energy of hydrogen in order for electrons to bind with the dominant form of matter. If, instead, helium-4 were the dominant form of matter, a different picture would develop. Helium-4 has a higher binding energy than hydrogen, and a photon would not have to lose as much energy before it dropped below the binding energy of helium-4. Therefore, electrons would be able to bind with helium-4 earlier on in the history of the universe, meaning that decoupling would happen earlier, decreasing the size of the surface of last scattering. Therefore, the amount of helium-4 produced in the fiery conflagration of the Big Bang should have an effect on the properties of a picture of the surface of last scattering, which is displayed in the cosmic microwave background radiation.

## ***2.2 The Cosmic Microwave Background***

A uniform background radiation in the microwave region of the electromagnetic spectrum was discovered by Arno Penzias and Robert Wilson in 1965 at Bell Laboratories in New Jersey. At first glance, they thought that it was noise created by the equipment they were

using to observe the sky. However, after repeated attempts to solve the noise problem, including removing pigeons, Penzias and Wilson determined that the signal was actually coming from the sky and not from the equipment, and that this signal had a temperature of about 2.7 Kelvin.

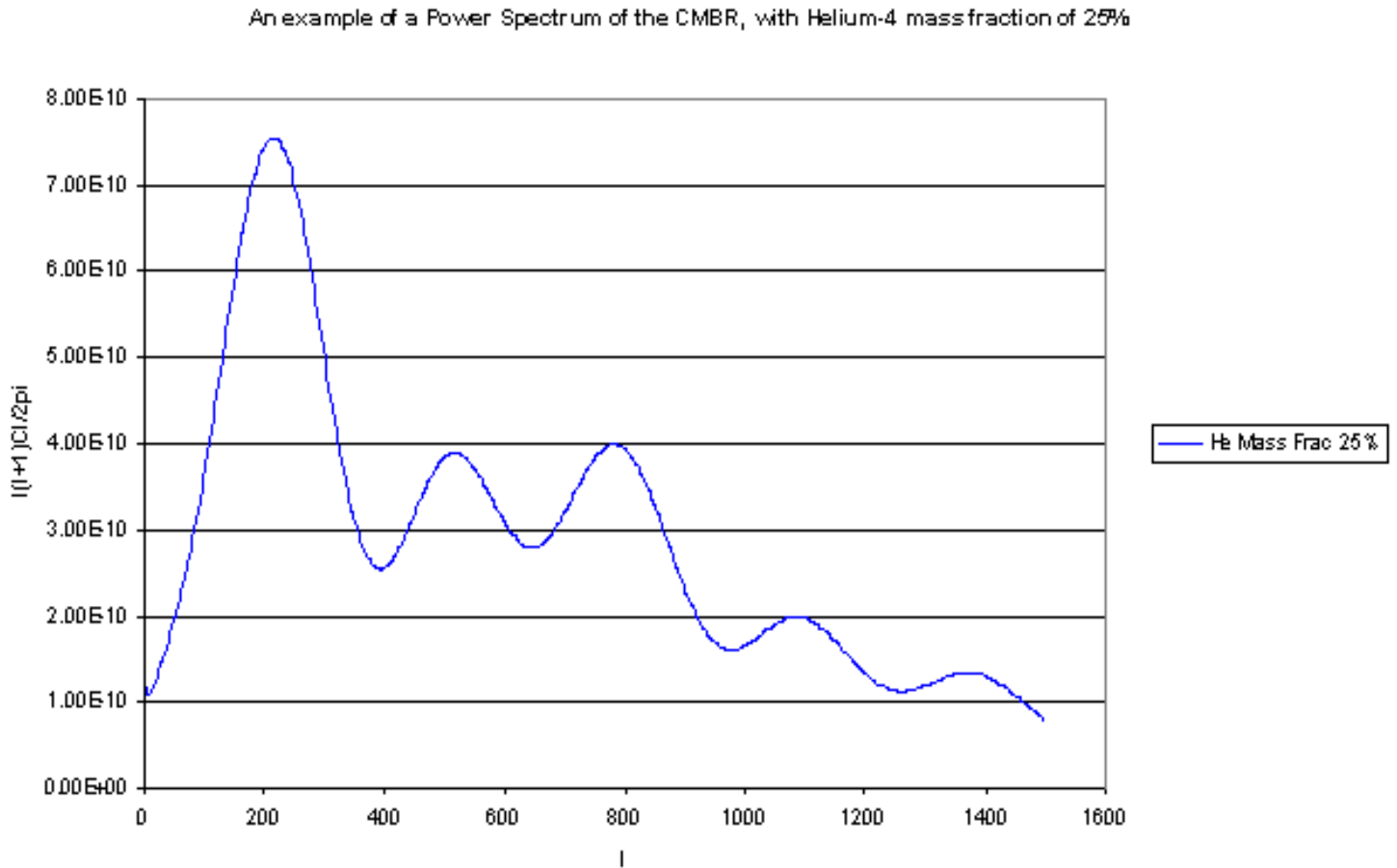
Unbeknownst to Penzias and Wilson, scientists had theorized that the Big Bang would have left a remnant that would be in microwave radiation and be around a temperature of 10 Kelvin. This experimental evidence earned Penzias and Wilson a Nobel Prize (Kosowsky, 2-3).

This radiation is called the cosmic microwave background radiation (CMBR). It is almost uniformly distributed across the entire sky, varying only on the order of  $10^{-5}$ . "Theorists quickly realized that fluctuations on its temperature would have fundamental significance as a reflection of the initial perturbations which grow into galaxies and clusters (of galaxies)" (Kosowsky, 4). If the CMBR were completely uniform, that would mean that the distribution of matter in the early universe was also completely uniform, and scientists would have a very hard time trying to determine how matter began to gravitationally come together to form things like galaxies. The CMBR can be looked at with the distribution of matter in the early universe in mind because it is basically an image of the surface of last scattering (Kosowsky, 7). The surface of last scattering is the term given to the picture of the universe at the time of recombination, or decoupling, which was described above. Recombination is a rather inaccurate term for the event, as the electrons and baryons stably combine for the first time to create neutral atoms.

The thermal properties of the surface of last scattering can be displayed as a graph through analysis of the CMBR, a graph called a power spectrum (Figure 1). The x-axis is the angular size, beginning at the order of a degree and decreasing in size. The y-axis is the

coefficient of the temperature fluctuations inherent in the CMBR, which actually are coefficients of spherical harmonics. Spherical harmonics are involved because the sky is roughly spherical

**Figure 1:** A sample power spectrum, with helium mass fraction of 0.25



and the fluctuations take on the form of oscillations due to the behavior of matter and radiation prior to decoupling. Before matter and radiation separated, photons had enough energy due to the temperature of the universe to keep electrons from binding to protons or neutrons, and essentially the baryons and the photons existed together in a fluid-like form. Areas of matter were attracted to other regions of matter by gravity, but the photons resisted being compressed. This sets up oscillations in the "fluid", which could be thought of as sound waves, since they were

longitudinal waves. For this reason, the peaks in the power spectrum are often referred to as acoustic peaks (Hu, 3). These oscillations are described by the following equation:

$$(m_{eff} \dot{\Theta}) + \frac{k^2}{3} \Theta = -\frac{k^2}{3} m_{eff} \Psi - (m_{eff} \dot{\Phi}),$$

which “is the fundamental relation for acoustic oscillations; it reads: the change in the momentum of the photon-baryon fluid is determined by a competition between the pressure restoring and the gravitational driving forces (Hu, 5)”.

By looking at the power spectrum of the CMBR, cosmologists can gain greater insight into the properties of the universe. One may ask how this is possible. It turns out that “the physical parameters determining the evolution of the initial perturbations until decoupling involve a few specific combinations of cosmological parameters” (Kosowsky, 26). First, the gravitational potentials that cause the matter to participate in the photon-matter effective spring are determined by the total matter density of the universe, in the form  $\Omega_0 h^2$ , where  $\Omega_0$  is the ratio of the total matter density of the universe to the critical density, and  $h$  is the dimensionless form of the Hubble constant  $H_0$ , where  $h = H_0/100 * (\text{Mpc*s})/\text{km}$ . As an aside, the shape of the universe can be represented by  $\Omega_0$ , where if it equals one the universe is flat, if it is greater than one, the universe is closed (which means that it will eventually collapse back in on itself), and if it is less than one, the universe is open, meaning it will expand forever. By looking at the peaks of the power spectrum, one can also have an idea of the baryon density in the same form as that of the total density,  $\Omega_B h^2$ . The baryon density has a specific effect on the nature of the power spectrum – different amounts will increase or decrease the height of the peaks. The total matter density and the cosmological constant are also important parameters in this, as they determine the distance to the surface of last scattering, which affects the placement of the curve on the angular scale along the x-axis (Kosowky 26-27). Since the baryon density of the universe and

the cosmological constant can be more closely determined by looking at the power spectrum of the CMBR, the dark matter density can also be more closely refined. This is due to the fact that the total matter density of the universe is the sum of the densities of baryons, dark matter, and the cosmological constant, and the total matter density is also more closely determined from the power spectrum. These are all important parameters in cosmology, leading those interested to a greater understanding of the universe.

### ***3. Purpose and Method***

The purpose of this project is to try to put constraints on the amount of Helium-4 that could have been produced during the fiery conflagration of the Big Bang, and, more fundamentally, if changing the value of the primordial helium-4 abundance has an effect on the power spectrum of the CMBR. Theory of nucleosynthesis during the Big Bang predicts the fraction of helium-4 by mass that should have been produced as roughly twenty-five percent. Analysis of the CMBR seems to support this mathematical derivation, but no one has used the CMBR power spectrum before to place constraints on the amount of helium-4.

#### ***3.1 CMBFAST***

A very useful computer program called CMBFAST was created by Ulrich Seljak and Matias Zaldarriaga with the strict intention of creating power spectra of the CMBR, after being supplied input parameters by the operator. It is a very complicated program with many different aspects. The input parameters that must be supplied to the program in order to successfully create a single power spectrum include the baryon density, the cold dark matter density (it runs on the assumption that the dark matter in the universe is cold), the vacuum energy density (cosmological constant), and the neutrino mass density, which has been taken to be zero. Also,

the Hubble constant must be entered, along with the temperature of the CMBR, the mass fraction of helium-4, the number of massless neutrinos, which has been taken to be 3.04, and the number of massive neutrinos, which was set to zero. Continuing with the necessary input, one must tell the program which of two versions of recombination to use. There exist two different subroutines for recombination in the program, one called “Peebles recombination”, which is named after the cosmologist P.J.E. Peebles, who developed the model, and another, more detailed version of the calculation, dubbed RECFAST, based on the work by Seager, Sasselov, and Scott. Whether or not reionization occurs must also be specified, along with the optical depth of the surface of last scattering if reionization is indeed included in the analysis. The type of perturbation must be entered, whether it be scalar or tensor, and the initial conditions that accompany it. These have been set, based on theory, to be scalar adiabatic perturbations. Finally, the scalar spectral index of the power spectrum to be created must be indicated, which basically is the tilt of the curve on the graph.

After getting adjusted to the program, the first method of exploration was to plot the power spectra created by keeping every parameter constant, except for the value of the mass fraction of helium, which would range from zero to one hundred. By doing this, it would be easy to tell if the amount of helium-4 taken as an initial condition would have an effect on the graph. Indeed, this was seen to be the case, for the curves produced were distinct. This was first done for the Peebles method of recombination. Theory predicted that the curves produced should “travel” down the graph with increasing percentages of helium. However, something very odd occurred when high values of helium were inputted. Once about ninety percent was reached, the curves started to behave unrealistically, traveling up the graph. The second method of recombination, RECFAST, was then employed to get a more detailed result. However, it was seen that this

method also produced erroneous results, with the curves jumping all over the graph once the range of seventy to eighty percent was reached.

It was theorized by Dr. Scherrer and Dr. Walker that there were two functions programmed into the complex program that could be causing the problem. These were the ionization fraction and the visibility function. The visibility function is represented by

$$g(z) = e^{-\tau(z)} \frac{d\tau}{dz},$$

where  $z$  is the redshift, and  $\tau$  is the optical depth of photons from Thomson scattering, described by

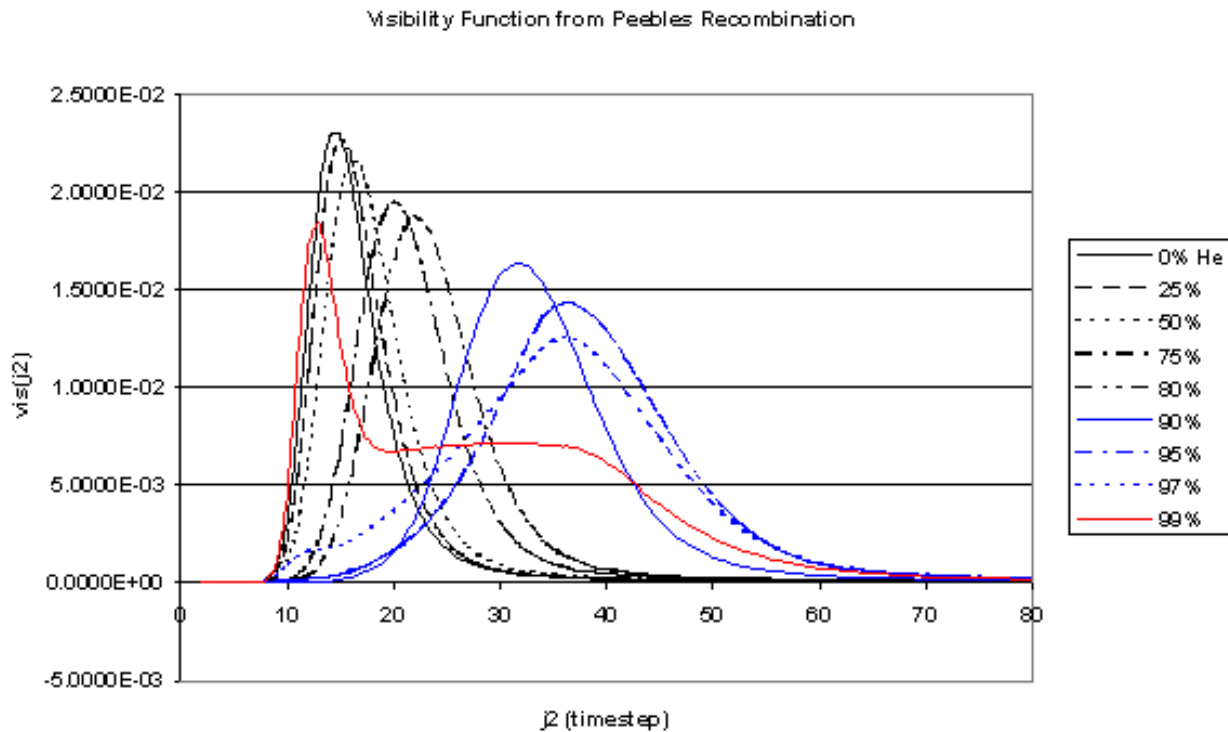
$$\dot{\tau} = x_e n_p c \sigma_T,$$

“where  $\sigma_T$  is the Thomson scattering cross-section,  $n_p$  is the number density of electrons (both free and bound) and  $x_e$  is the ionization fraction” (Kaplinghat, et. al. 2-3). The visibility function describes the conditions when the universe became transparent to light (recombination). The ionization fraction  $x_e$  is a function that describes how hydrogen and helium were ionized by the energetic photons that they encountered prior to the formation of neutral atoms. It is proportional to the quantity  $z/\alpha^2$ , where  $z$  is redshift and  $\alpha$  is the fine structure constant (Kaplinghat 3).

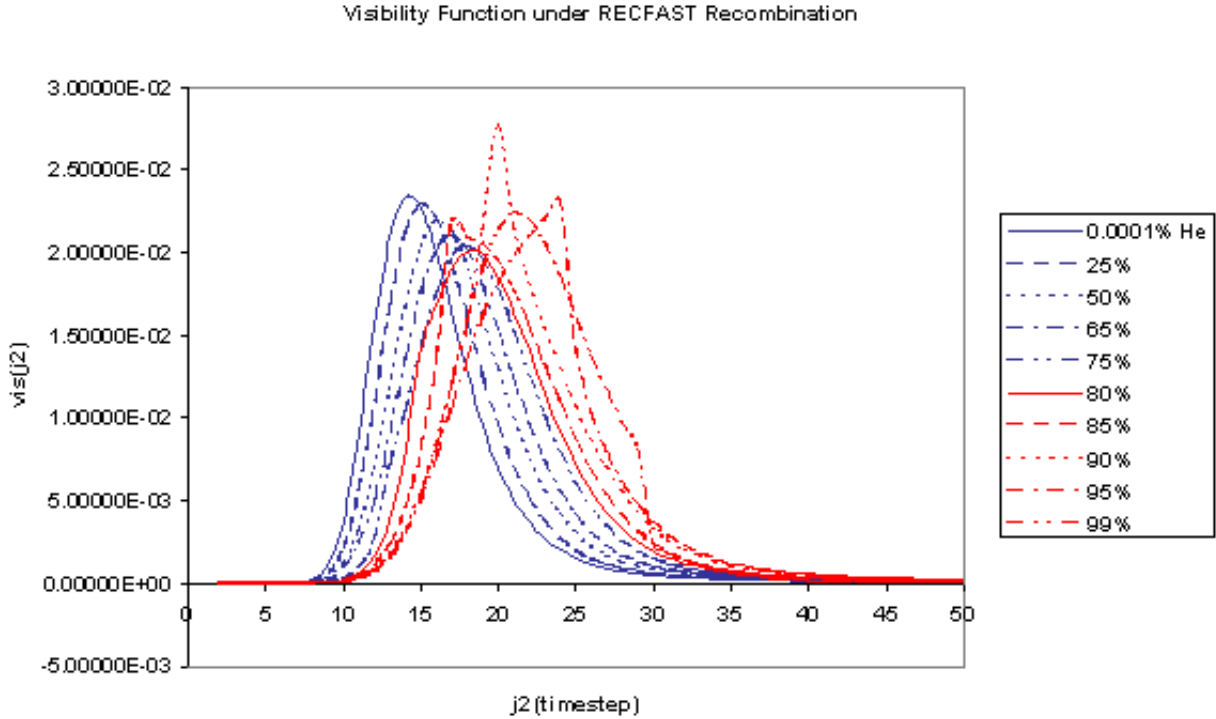
After exploring the FORTRAN code for CMBFAST, the ionization function was found and the code was modified to print out the values of the function, which varied with time. Theoretically, the resulting plotted curves should be smooth, both in form and in the transition from one different mass fraction of helium-4 to another. This was indeed found to be the case, and so the ionization fraction was disregarded as a problem. Next, the visibility function was found and plotted, again as a function of a timestep. Very odd behavior was observed once those

curves were plotted. Under the Peebles method of recombination, the curves began acting strangely around a helium mass fraction of eighty-five percent, up through one hundred percent, paralleling the unrealistic behavior in the power spectra produced for those values of helium abundance. The RECFAST version of recombination was then looked at, and the visibility function showed weird behavior at eighty-five percent and possibly less, suggesting that the visibility function, too, was responsible for the erratic behavior of the power spectra produced with RECFAST (See Figures 2 and 3). It was then decided that only the Peebles recombination method would be used, and only up to values where it was still valid. Values of the mass fraction of helium of ninety percent and above are very unphysical, and can be disregarded.

**Figure 2:** The visibility function found by using the Peebles method of recombination



**Figure 3:** The visibility function as found by using the RECFAST method of recombination



Due to the fact that a number of parameters could be varied in the exploration of power spectra of the CMBR, it was necessary to explore all the variations. It is possible to raise one parameter while lowering another and reproduce a curve that resembles one with a completely different value for the helium mass fraction. The parameters that needed to vary were the baryon density  $\Omega_B$ , the cold dark matter density  $\Omega_C$ , the vacuum energy density (cosmological constant)  $\Omega_\Lambda$ , the Hubble constant  $H_0$ , the spatial spectral index (tilt)  $n_s$ , the optical depth for reionization  $\tau_c$  and, of course, the mass fraction of helium  $Y_{He}$ . A computer program was written in FORTRAN to draw random numbers from a forced Gaussian distribution for the five parameters besides helium, and models using these random numbers were made, to display a band of curves for each value of the mass fraction of helium. The central values of the parameters and the uncertainty associated with each from which the Gaussian distributions were created are

displayed in the table below. The parameters that were taken to be fixed were the density associated with neutrinos (0), the temperature of the CMBR (2.726 K), and the number of massless neutrinos (3.04).

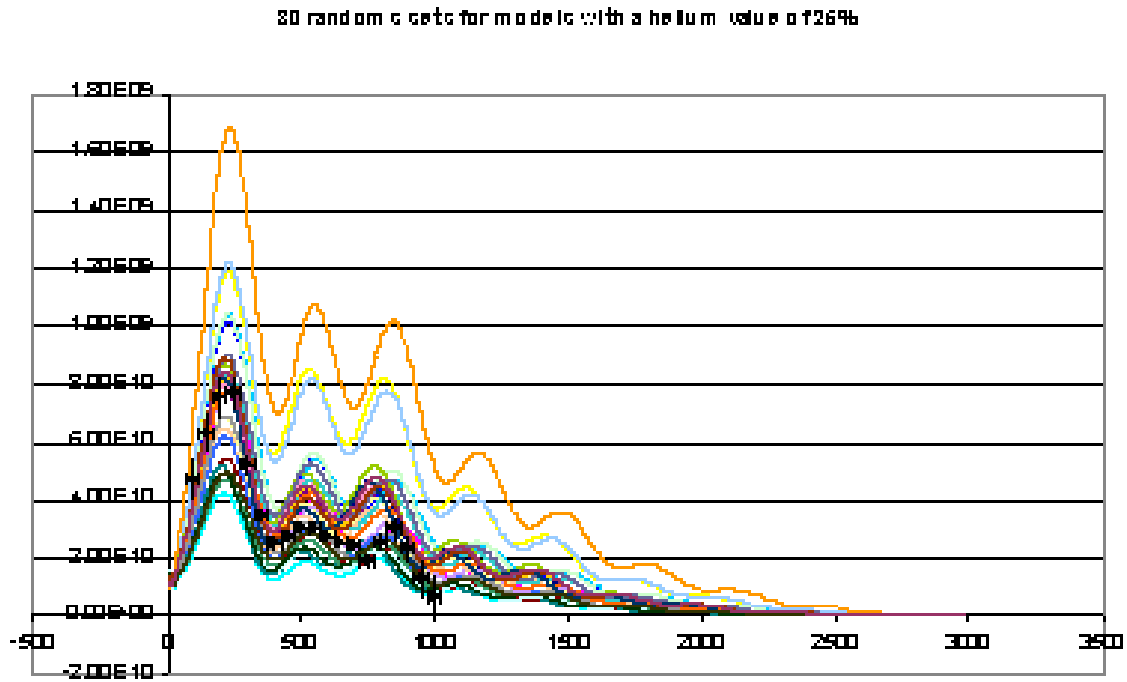
Table 1: Values of the varying parameters from which random numbers were drawn

Parameter	$\Omega_B$	$\Omega_\Lambda$	$\Omega_C$	$H_0$	$n_s$	$\tau_c$
Central Value	0.05	0.66	To make universe flat	66	1.03	0.15
Uncertainty	0.01	0.05	-----	5	0.1	0.13

These values are taken from the "concordance model" for the universe from the BOOMERANG data paper (Netterfield, et. al. 10), and come from assumptions based on the universe being flat, the existence of large scale structure, and the data from type Ia supernovae.

This was done in order to compare these theoretical curves to the experimental data collected by the BOOMERANG project, to see if values could be found that definitely did not fit the data and could be used as constraints on the amount of helium that could have been produced in the early universe. The resulting band of curves covers a region with a sparse area and a dense area. In order to determine the "fitness" of a given set of curves, one must look at the dense region, and at about sixty-six percent of the curves surrounding the centroid, which would be inside the dense region. Plots were made for random numbers drawn from the Gaussian distribution for values of the helium abundance that could possibly be constraining values when compared to the actual experimental data gathered by the BOOMERANG project. A sample plot of a set of random models is shown in Figure 4.

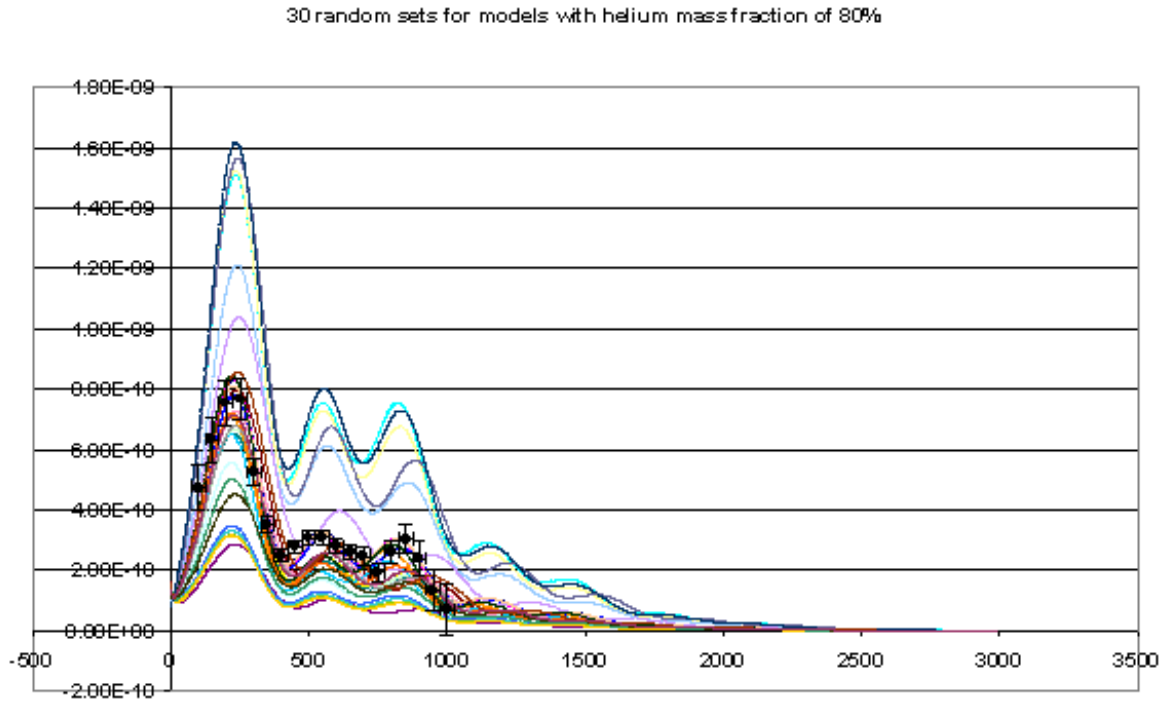
**Figure 4:** A sample graph of the random modeling, with a helium value of 25%. The data points are from BOOMERANG



#### ***4. Results***

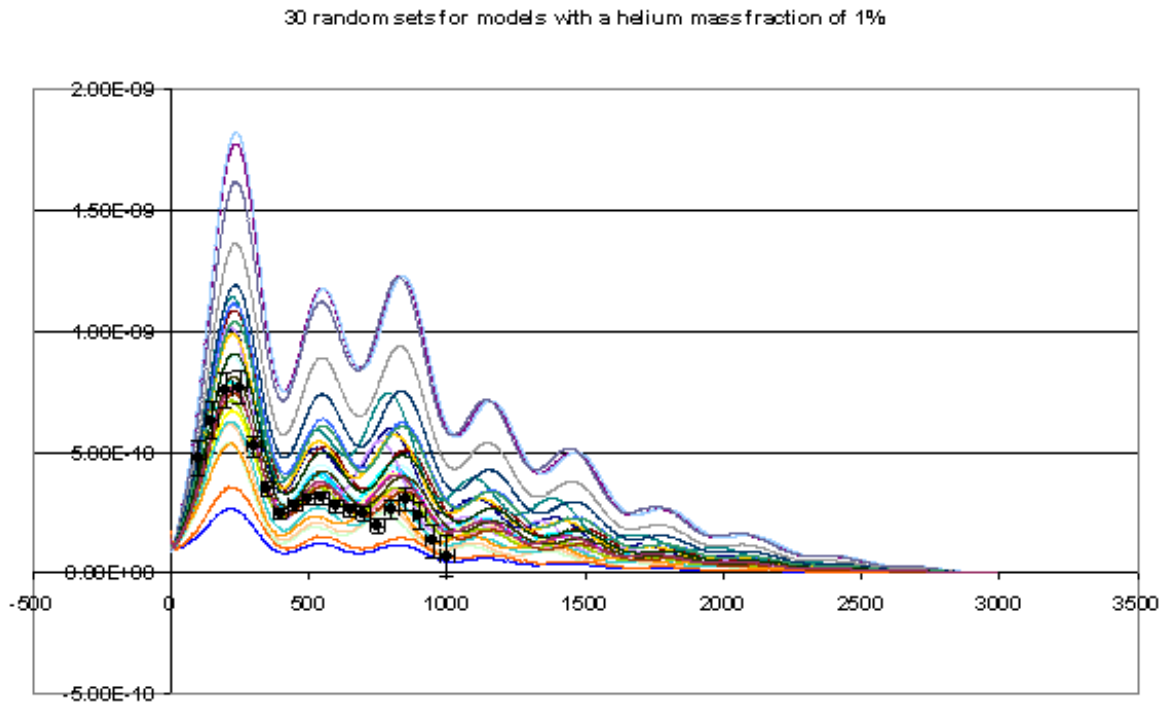
After looking at the different plots, it was determined that a value of eighty percent for the mass fraction of helium could confidently be placed as the upper boundary on the system, meaning that everything above eighty percent could be ruled out. This is because sixty-six percent of the curves can be found to lie beneath the data points, showing it to be unfavorable. Upon further analysis, a lower boundary could not be confidently chosen. The helium mass fraction of one percent could not be conclusively ruled out according to the criteria used for the study. However, that does not mean that a lower bound does not exist, just that it cannot be found by the methods employed in this project. See Figures 5 and 6.

**Figure 5:** The graph of the 30 random sets for a helium value of 80%. The data points with error bars are



from the BOOMERANG project.

**Figure 6:** The graph of 30 random sets for a helium value of 1%. The data points with error bars are from the BOOMERANG project.



## 5. *Further Research*

The method that was used to scan over all the parameters is rather crude in comparison to other statistical techniques. Further research for this project will include looking into the Fisher Information Matrix, a powerful statistical tool that can take a function or system that depends on many different varying parameters and can find the range that has the best fit values for those parameters. It also assumes that certain parameters are known with unlimited accuracy. It is found specifically for the CMBR power spectrum through

$$F_{ij} = \sum_{l=2}^{l_{\max}} \frac{1}{\Delta C_l^2} \frac{\partial C_l}{\partial \theta_i} \frac{\partial C_l}{\partial \theta_j},$$

where  $C_l$  is a coefficient of the spherical harmonics of a power spectrum, and  $\theta_i$  and  $\theta_j$  are varying parameters. The derivatives are calculated by two-sided finite differencing, meaning the value is measured at the high end of its range and at the low end of its range (Kaplinghat 7). This method will be more accurate, producing a narrower band for each set of models, hopefully allowing for a much tighter constraint on the value of the primordial helium-4 abundance.

## 6. *Conclusions*

The purpose of this research project dealt with the effect of the primordial abundance of helium-4 on the power spectrum of the cosmic microwave background radiation. First, it was to be determined if a change in the abundance of primordial helium-4 had an observable effect on the power spectrum of the CMBR, using a computer program called CMBFAST. For example, could one see a difference in a power spectrum with one percent mass fraction of helium-4 and one with a sixty percent mass fraction of helium-4? The answer is yes, and the effect is very noticeable, especially at higher values of the angular size. Next, the goal was to see if upper and

lower constraints could be placed on the amount of helium-4 produced during the Big Bang, given a set of varying parameters. This was explored through plotting sets of curves for specific values of the mass fraction of helium-4 while drawing on Gaussian distributions for random numbers of the varying parameters: the baryon density, the cold dark matter density, the vacuum energy density (totaling one so the universe is flat), the Hubble constant, the spatial spectral index (tilt of the curve), and the optical depth. A confident upper limit was concluded to be an amount of eighty percent for the percentage of the total mass of the universe contained in helium-4 produced from the Big Bang. Unfortunately, no conclusive lower limit could be discerned, although one could very well still exist and is only undetectable by the methods used in this project. Further research will be conducted to see if tighter constraints can be made on the primordial abundance of helium-4 using the Fisher Information Matrix, which is a more accurate statistical technique.

## ***7. Acknowledgments***

I would like to thank my advisor, Dr. Terry Walker, and my collaborator on this project REU student David Johnston from Brigham Young University, along with David's advisor Dr. Robert Scherrer. Both Dr. Scherrer and Dr. Walker were very helpful in providing insight and direction for the project, and it was a pleasure working with them. Also, I would like to thank Dr. Richard Kass of the High Energy Physics group at the Ohio State University for his help with FORTRAN. Finally, I would like to thank Dr. William Palmer for selecting me to be in Ohio State University's REU program for 2001 and the National Science Foundation.

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*Appendix A:*

Computer program to draw random numbers from a Gaussian distribution.

```

c file: random2.f
c Program to generate Gaussian random numbers for the
c cosmological parameters omegab, omegal, H0, n_s, t_c and omegac,
c which is not randomly formed but rather is 1-omegab-omegal
  program random
  implicit none
  integer seed, n, nmax
  real pi, omegab, omegal, omegac, n_s, H0, Y_He, G1, G2, G3, G4
  real r1, r2, r3, r4, r5, t_c, G5
  parameter (pi=3.14159)
c omegab is the baryon density, omegac is the cold dark matter density,
c omegal is the vacuum energy density, H0 is the Hubble constant,
c n_s is the tilt of the power spectrum, and Y_He is the
c primordial abundance of Helium-4
c
c Input a seed (large odd integer) to start the random process
  write(*,*) 'seed='
  read(*,*) seed
c Input a value for the primordial abundance of Helium-4
  write(*,*) 'Y_He='
  read(*,*) Y_He
c Input the number of sets to run the program for
  write(*,*) 'number of sets to run'
  read(*,*) nmax
  do n=1, nmax
c
c get a gaussian random number through the formula below, with the random
c number as the variable
c Transform the gaussian random number, which has a mean of 0 and a
c standard deviation of 1, to the random number you want, with a
c mean of M and a standard deviation of S by using
c Value=S*G + M
      call omegabsub(seed, omegab)
      write(*,*) 'omegab ', omegab
      call omegalsub(seed, omegal)
      write(*,*) 'omegal ', omegal
      omegac=1-omegab-omegal
      write(*,*) 'omegac ', omegac
      call H0sub(seed, H0)
      write(*,*) 'H0 ', H0
      call n_ssub(seed, n_s)
      write(*,*) 'n_s ', n_s
      call reionizsub(seed, t_c)
      write(*,*) 't_c', t_c
      write(*,*) 'Y_He ', Y_He
c
c write it to the file fort.21, with format of 3 decimal places for
c omegab, omegac, and omegal, 1 decimal place for H0, and 2 decimal
c places for n_s and Y_He
      write(21,100) omegab, omegac, omegal, H0, n_s, t_c
100    format(1x'omegab 'f6.3,1x'omegac 'f6.3,1x,'omegal 'f6.3,
+      1x,'H0 'f6.1,1x,'n_s 'f6.2,1x,'t_c 'f6.2)
c 100    format(f6.3, 1x f6.3, 1x, f6.3, 1x, f6.1, 1x, f6.2, 1x, f6.2)

  enddo

```

```

stop
end

subroutine omegabsub(seed, omegab)
implicit none
integer seed, n, nmax
real pi, omegab, r1, G1
parameter (pi=3.14159)
r1=ran(seed)
G1=sqrt(-2.0*log(r1))*cos(2*pi*r1)
omegab=0.01*G1+0.05
c   write(*,*) 'r1 ', r1
return
end

subroutine omegalsub(seed, omegal)
implicit none
integer seed, n, nmax
real pi, omegal, r2, G2
parameter (pi=3.14159)
r2=ran(seed)
G2=sqrt(-2.0*log(r2))*cos(2*pi*r2)
omegal=0.05*G2+0.66
c   write(*,*) 'r2 ', r2
return
end

c   subroutine omegacsub(omegab, omegal)
c   implicit none
c   real omegab, omegal, omegac
c   omegac=1-omegab-omegal
c   return
c   end

subroutine H0sub(seed, H0)
implicit none
integer seed, n, nmax
real pi, H0, r3, G3
parameter (pi=3.14159)
r3=ran(seed)
G3=sqrt(-2.0*log(r3))*cos(2*pi*r3)
H0=5*G3 + 66
c   write(*,*) 'r3 ', r3
return
end

subroutine n_ssub(seed, n_s)
implicit none
integer seed, n, nmax
real pi, n_s, r4, G4
parameter (pi=3.14159)
r4=ran(seed)
G4=sqrt(-2.0*log(r4))*cos(2*pi*r4)
n_s=0.1*G4+1.03
c   write(*,*) 'r4 ', r4
return

```

```
end

subroutine reionizsub(seed, t_c)
implicit none
integer seed, n, nmax
real pi, t_c, r5, G5
parameter (pi=3.14159)
r5=ran(seed)
G5=sqrt(-2.0*log(r5))*cos(2*pi*r5)
t_c=0.13*G5+0.15
return
end
```