Electron Beam Harmonic Generation of Submillimeter Waves

Amy Sommer

University of Wisconsin-Parkside

Prof. Frank De Lucia, Ph.D.

The Ohio State University

Research Experience for Undergraduates
Summer 2001

1 Introduction

For many years Klystrons have been used as a way to produce power. Currently klystrons are used in many accelerators. For example, SLAC's Next Linear Collider will have approximately 10,000 klystrons. To produce power the current klystrons make use of only the first order harmonic of the produced current. Making use of higher order harmonics to generate submillimeter waves is the goal of this project. The first calculations are based on the preexisting klystron. This is used as a starting point for all other calculations. To begin, the current in the klystron is calculated. To determine the harmonic content of the current a Fourier Transformation is employed. Then similar calculations are performed after an accelerating voltage is added to the current klystron. This is the first attempt at making use of higher order harmonics. However, many further calculations must be made to get to the ultimate goal of the project, to construct a device that would be able to generate submillimeter waves using higher order harmonics.

2 Klystron

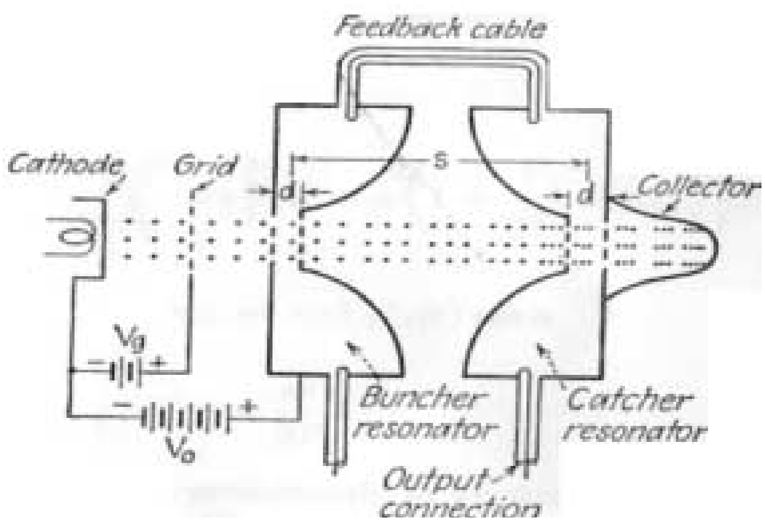


Figure 1 – Klystron Oscillator [1]

[1] Figure from Theory and Application of Microwaves

2.1 How a klystron works

A beam of electrons is accelerated by applying a d-c voltage (V_0). The electrons then enter the buncher with a velocity v_0, which can be determined using the kinetic energy of the electrons as they enter the buncher:

$$\frac{1}{2} m v_0^2 = eV_0$$

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

As the electrons pass through the buncher their velocities are altered, some are accelerated while others are decelerated. This occurs when they pass through the alternating potential $V_1 \mathrm{Sin}(\omega t)$. After the electrons pass through the potential they drift through the cavity and form bunches. These bunched electrons then pass through the catcher, which works much like the buncher. The catcher, also containing an alternating potential, removes energy from the bunched electrons by decelerating them. As the bunched electrons pass through the catcher the electric field induced by the alternating potential slows down the bunched electrons. Since the velocity of the electrons decreases, the energy of the electrons must also decrease. This energy is then transferred to the output cavity.

2.2 Bunching Electrons/Velocity Modulation

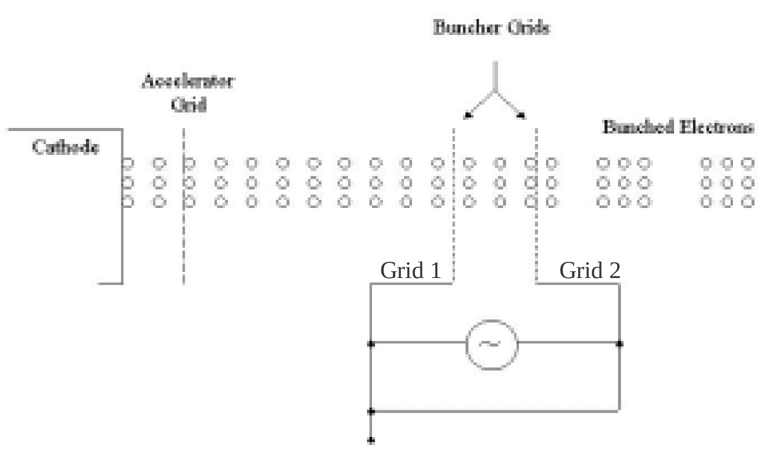


Figure 1 -- Buncher Grids

As the electrons pass through the buncher grids they are either accelerated or decelerated, depending on the direction of the electric field. While the electric field, induced by the alternating voltage $V_1 \sin \omega t$, points in the same direction as the movement of the electrons, the electrons will be decelerated. If grid 1, in Figure 2, is positively charged while grid 2 is negatively charged, the negatively charged electron is decelerated since the direction of the electric field is opposite of the direction of travel of the electron. The opposite is also true. While the electric field points in the same direction as the movement of the electrons, they are accelerated. This occurs if grid 1 is negatively charged while grid 2 is positively charged. While the electron's velocity is decreased the electron loses energy to the electric field and while it accelerates the electron acquires extra energy from the electric field. The degree in which the electron is accelerated or decelerated depends on the time at which it passes through the buncher grids. If an electron passes through the buncher when ωt_1 is an integer multiple of $\frac{\pi}{2}$, then the electron is at the maximum velocity of $v_0\sqrt{1+V_1/V_0}$. Also when ωt_1 is an integer multiple of $-\frac{\pi}{2}$, the electron's velocity is at the minimum of $v_0\sqrt{1-V_1/V_0}$. The velocities of all other electrons vary between the maximum and minimum, depending on the time they pass through the buncher. As the electrons exit the buncher and travel through the drift cavity they form bunches and disperse continuously, as seen in Figure 3, until acted upon by another force. This is due to the variation in individual electron velocities. Each line in Figure 3 represents the motion of one electron with the electron's velocity proportional to the slope of the line. The distance the electron travels in the drift space as a function of time is

$$d = v_0(t - t_0)\sqrt{1 + \frac{V_1}{V_0} Sin\omega t_1}$$

where t_1 is the time the electron leaves the buncher grid.

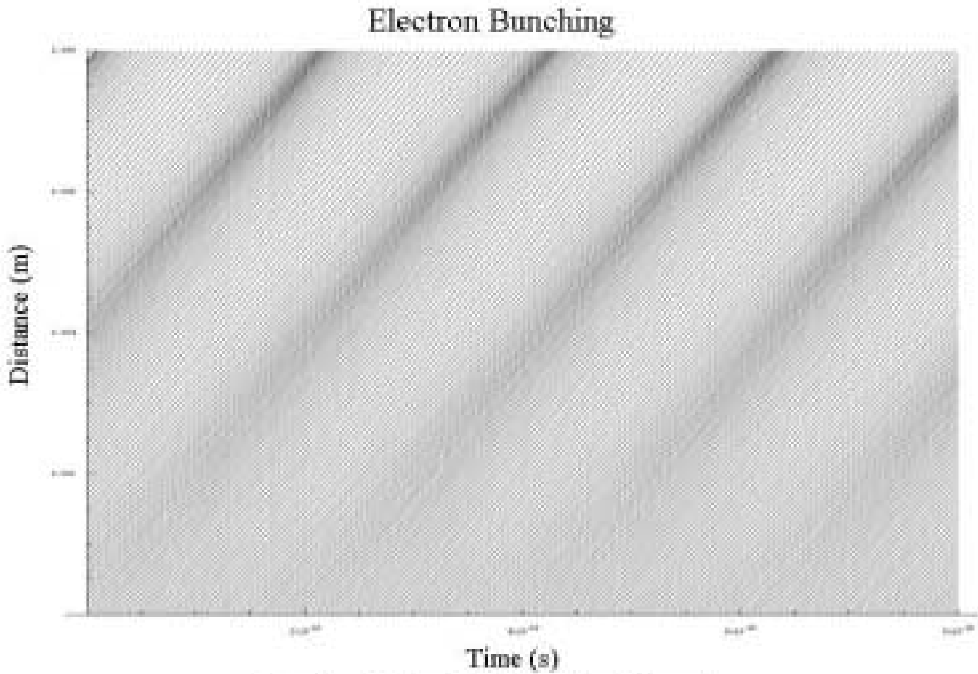


Figure 3 – Electron Bunching in the drift cavity

2.3 The Current in the Drift Cavity

The electron bunching generates the current, in the drift cavity at a given distance. To obtain the current, one must recall that current is a measure of the rate of charge flow, so $I_i = \frac{\Delta q}{\Delta t_i}$. To calculate the current at the end of the drift space as the electron beam enters the catcher, the time T that it takes an electron to travel from the buncher to the catcher grid must be taken into consideration. The equation is the following

$$T = \frac{s}{v_0 \sqrt{1 + \frac{V_1}{V_0} Sin\omega t_1}}$$

where s is the distance from the buncher grid to the catcher grid. If the electron reaches the catcher at time t_2 then $t_2 = t_1 + T$. Therefore

$$I_2 = \frac{\Delta q}{\Delta t_2} \longrightarrow I_2 = \frac{\Delta q}{\Delta t_1 (1 + \frac{\Delta T}{\Delta t_1})} \longrightarrow I_2 = \frac{I_1}{1 + \frac{\Delta T}{\Delta t_1}}$$

where I_1 is the current at the buncher grid. By differentiating T with respect to t_1 you get

$$\frac{\Delta T}{\Delta t_1} = \frac{-s\omega V_1 Cos\omega t_1}{2v_0 V_0 (1 + \frac{V_1}{V_0} Sin\omega t_1)^{3/2}}$$

Plugging this back into the equation for the current, finally the current at the entrance to the catcher becomes

$$I_2 = \frac{I_1}{1 - \frac{s\omega V_1}{2v_0 V_0}(1 + \frac{V_1}{V_0} Sin\omega t_1)^{-3/2} Cos\omega t_1}$$

Clearly the current is a sinusoidal function, as seen in Figure 4, due to the bunching of electrons. While the electrons are bunched, the current is large and so it follows that when there is a “gap” of electrons, the current decreases. Figure 4 also demonstrates that the current is a periodic function with a period of $2\pi/\omega$.

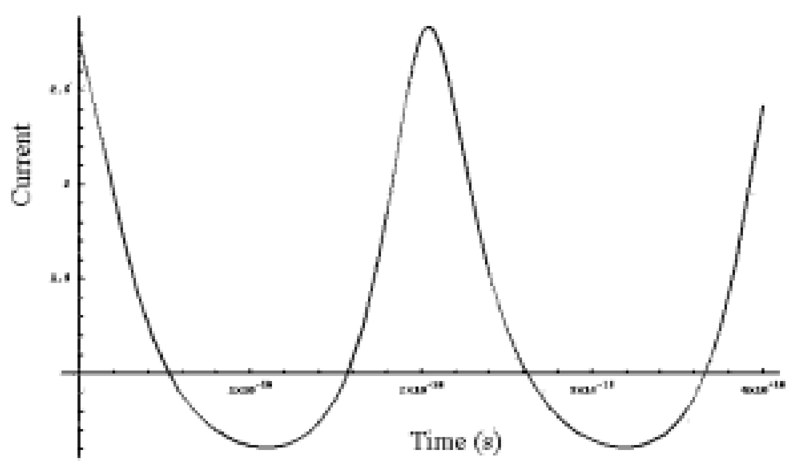


Figure 4 – Current in a Klystron at the End of the Drift Cavity

3 Modification of the Klystron

3.1 Fourier Analysis of the Current in a Klystron

In typical calculations of the current it is assumed that V_1/V_0 is very much less than one therefore only the first two terms in the binomial expansion of

$$T = \frac{s}{v_0\sqrt{1+\frac{V_1}{V_0}Sin\omega t_1}}$$

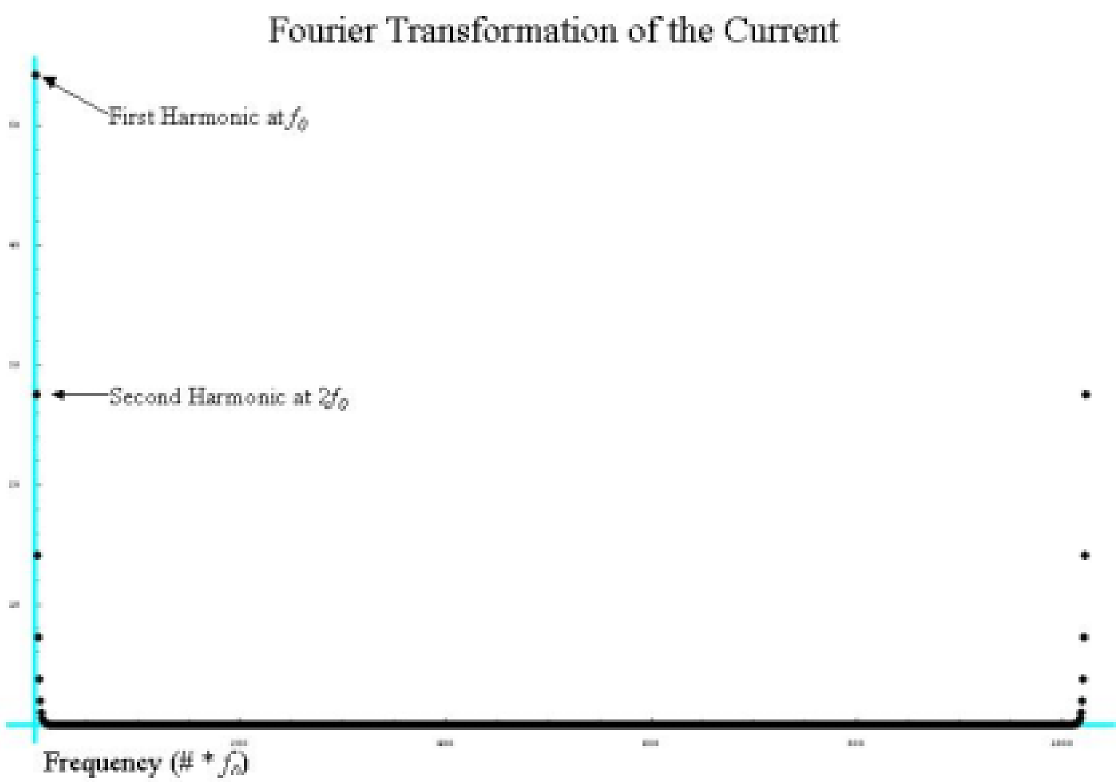
are taken, assuming that all other terms are negligible. This leaves

$$T = \frac{s}{v_0}(1-\frac{V_1}{2V_0}Sin\omega t_1)$$

Making the current at the entrance of the catcher grid

$$I_2 = \frac{I_1}{1-\frac{sV_1\omega}{v_0 2V_0}Cos\omega t_1}$$

Taking this approximation of the current only the first order harmonic of the current is taken into consideration. To find the second order and higher harmonics of the current, a Fourier transformation of the current without the approximations is taken. Below is the Fourier transformation of the current plotted in Figure 4.



To interpret the graph of the Fourier transformation, one must have some understanding of what a Fourier transformation is. A Fourier transformation takes a function and decomposes it into a sum of simple sinusoidal waves of different amplitude. When plotting the Fourier transform, as in Figure 5, the x-axis is frequency and the y-axis is amplitude. When viewing the Fourier transformation plot it is important to realize that the second half of the graph is a mirror image of the first half. The only point that is not reflected is the first point corresponding to the fundamental frequency. Every point on the x-axis represents a frequency; point one corresponds to f_0 while the second point corresponds to 2 f_0 and so on. As seen in Figure 5, the harmonics of the frequencies rapidly decrease in amplitude. This is true because the amplitude of the first harmonic is on the order of $\frac{V_1}{V_0}$, while the order of the second harmonic is on the order of $\left(\frac{V_1}{V_0}\right)^2$ and so on following with the third and higher harmonics. Taking the binomial expansion of the current and keeping more than the first term shows the dependence on v/v. Below is the binomial expansion keeping the first three terms.

$$I_2 = \frac{I_0}{1 - \frac{s\omega}{2v_0}\left[\frac{V_1}{V_0}Cos\omega t_1\left(1 + \frac{15}{32}\left(\frac{V_1}{V_0}\right)^2\right) - \left(\frac{V_1}{V_0}\right)^2 Sin2\omega t_1 \frac{3}{4} - \left(\frac{V_1}{V_0}\right)^3 Cos3\omega t_1 \frac{15}{32}\right]}$$

3.2 Addition of an Accelerating Voltage

To begin the attempt of using higher order harmonics to generate submillimeter waves, the klystron is modified. The first modification is the addition of an accelerating voltage that is applied after the electrons are allowed to coast in the drift cavity for a short distance. In the first section of the drift cavity there is no potential difference, so the electrons are allowed to travel through the cavity at constant velocity. The addition of the accelerating voltage is a means of changing the current in hopes that the higher order harmonics will be more prevalent than in the existing klystron.

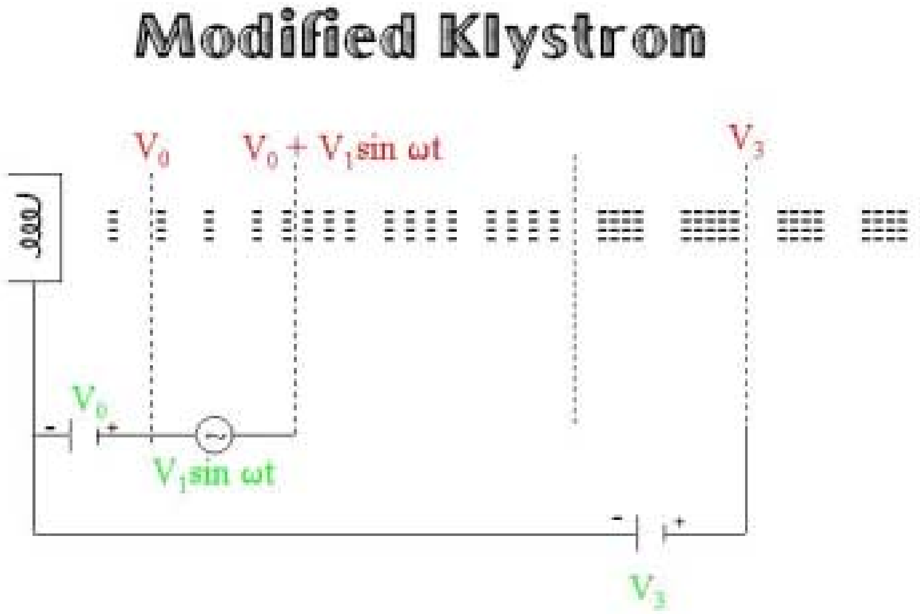


Figure 6 – Typical Klystron with the addition of an accelerating voltage

3.2.1 Further Electron Bunching

With the addition of an accelerating voltage, the already bunched electrons are accelerated through the remainder of the cavity. To determine the amount of further bunching, the equation for the distance that a single electron travels while under the influence of the accelerating voltage must be obtained. To determine this, one must take into consideration the force experienced by the electron within the second part of the drift cavity. The second part of the drift cavity contains the accelerating voltage, while there is no potential difference within the first part of the drift cavity. Therefore, the force experienced by the electrons is eV_3 / s_1 where s_1 is the distance measured from the beginning of the second part of the drift cavity. To determine the equation for the distance one must use $F = ma$ to obtain the equation for acceleration,

$$\frac{d^2x}{dt^2} = \frac{eV_3}{ms_1}$$

If the electron enters the second part of the drift cavity at time $t = t_2$ with an initial velocity of v_1 then one integration leads to the equation for the velocity,

$$v = v_1 + \frac{eV_3}{ms_1}(t - t_2)$$

With the electron's initial position s, the distance from the buncher grid to the beginning of the second part of the drift cavity, one more integration leads to the equation for distance,

$$x = s + v_1(t - t_2) + \tfrac{1}{2}\frac{eV_3}{ms_1}(t - t_2)^2$$

Using the above equation, one can make a plot of the distance the electrons travel over time. Figure 7 demonstrates the bunching that occurs within both sections of

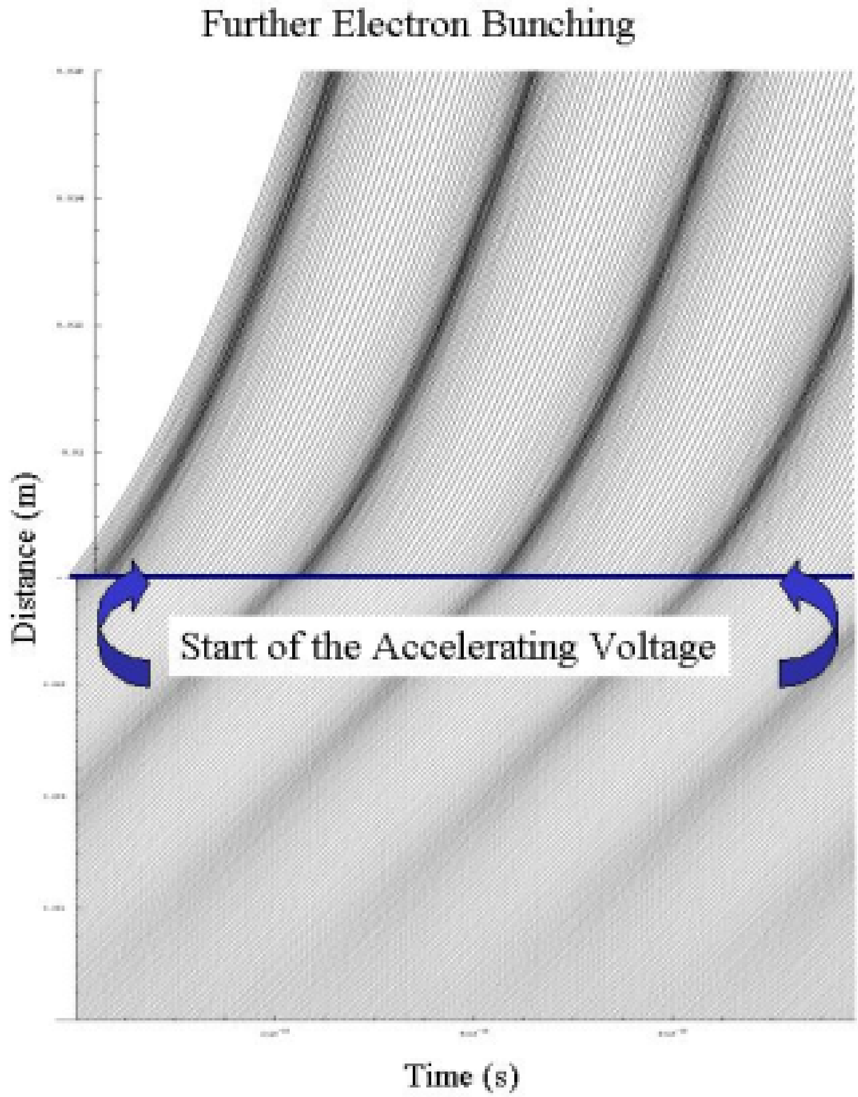


Figure 7 – Electron bunching with an added accelerating voltage

the drift cavity. The bottom half of the graph represents what happens in the first section of the drift cavity where there is no potential difference to influence the velocity of the electrons. The top half of the graph represents the second section of the drift cavity. Once the accelerating voltage is applied to the electrons, the rate at which the electron bunching occurs is much faster.

4 Conclusions

4.1 Further Calculations to be Made

The above listed calculations are just a minor step in the long process of making use of higher order harmonics to generate submillimeter waves. Many further calculations must be made to determine the exact methods to do so. The first calculation that must be made is calculating the current with the addition of the acceleration voltage. Then taking the Fourier transformation of the current in the second part of the drift cavity must be done. It is necessary to do so to find the harmonics of the current, as done for the current in the first cavity, the typical klystron. Another factor is to determine by what parameter the harmonics vary. In the case of the ordinary klystron it is seen that the harmonics vary according to powers of $\frac{V_1}{V_0}$. Other important calculations that must be performed include things such as determining the optimal distance that the electrons should travel in the first part of the drift cavity. The various voltages (V_0, V_1, V_2) must also be determined, depending on the values that generate the greatest amount of bunching. The distance the electrons travel in the second section of the drift cavity must also be calculated to optimize the amount of bunching produced. These are just a few of the many further calculations that must be made to even begin to consider the construction of such a device. The part of this project that I have worked on is just the first step to creating a device that uses higher order harmonics to generate submillimeter waves.

5 Acknowledgements

I would like to give special thanks to my mentor Frank De Lucia, Ph.D. for guiding me through my project for the past ten weeks. I look forward to continuing my work with him on this project. I would also like to thank Tom Lemberger and William Palmer for all of their time and for making this an enjoyable program. Another thanks to Tom Kelch for the shop tutorial, I had a most enjoyable time learning to use the machinery. Thanks to Shirley Royer for the excellent job she has done. Most of all I would like to thank the National Science Foundation and The Ohio State University for sponsoring the program.

6 References

1. Theory and Application of Microwaves, Arthur B. Bonwell, M.S., M.B.A., McGraw –Hill Book Company, Inc., New York & London, 1947