PHYSICS 846

Home Work Assignment # 4 2/10/2009

Due: **Tues., Feb. 17, 2009**.

1. *Classical and Quantum harmonic oscillators:* Work through the solutions of Kardar, Problems 1 and 2 of Chapter 4 (page 256 - 260). Compare the canonical ensemble derivation done in class with the microcanonical version in these problems.

2. *Hard sphere gas:* Work through the solution of Kardar, Problem 4 of Chapter 4 (page 262).

3. *Molecular Oxygen:* Kardar, Problem 10 of Chapter 4 (page 124). In parts (b) and (c), also find the low and high temperature limits and comment on the results.

4. *Ortho/Para Hydrogen:* Kardar, Problem 9 of Chapter 6 (page 179).
Hints: (i) Keep track of the degeneracies of various energy levels
(ii) Argue that only a few terms dominate Z in the low temperature limit. Keep the leading non-trivial (T-dependent) term.
(iii) Argue that contributions from large quantum numbers dominate Z in the high temperature limit. The resulting sum can then be approximated by an integral of the form $\int_0^\infty dx x \exp(-x^2)$.

5. Bohr-van Leeuwen Theorem on the *impossibility of magnetism in classical physics*:
Consider a classical N-particle system with *arbitrary, multiparticle interactions*, described by the Hamiltonian:

$$\mathcal{H} = \sum_{j=1}^{N} \frac{\mathbf{p}_j^2}{2m} + \mathcal{V}(\mathbf{r}_1, \ldots, \mathbf{r}_N)$$

in a box of volume V in thermal equilibrium at a temperature T. An external magnetic field $\mathbf{H}$ then modifies this Hamiltonian as follows. Define the vector potential $\mathbf{A}$ via $\mathbf{H} = \nabla \times \mathbf{A}$, and make the replacement $\mathbf{p}_j \to (\mathbf{p}_j - e\mathbf{A}/c)$ in the kinetic energy term of $\mathcal{H}$ without any change in the potential energy.

To see if the system has any (non-trivial) magnetic behavior, write down the classical partition function $Z(T, N, V, \mathbf{H})$. Use a change of variables to show that this is identical with $Z(T, N, V, \mathbf{H} = 0)$. Hence argue that the classical free energy is independent of the external field $\mathbf{H}$, and thus the magnetization $\mathbf{M}$ is identically zero.

6. Consider thermal radiation at a temperature T in a cavity of volume V.
(a) Find the *number density* of photons.
(b) Find the *entropy density* of photons.
(c) What is the *average entropy per photon*? (Please give a numerical value to at least three significant digits for the 'constant' prefactor).
(d) Show that the radiation pressure is one third the energy density: $p = U/3V$.

7. Observations of the cosmic microwave background radiation are of great importance in cosmology. The goal of this problem is to show that in an expanding universe, the temperature used to characterize the radiation decreases as the universe expands. Hence the radiation originating in the big bang is now at approximately 3 K.

The *energy per unit volume du* of blackbody radiation at a temperature T in a narrow interval of wavelengths from λ to $\lambda + d\lambda$ can be written as

$$du = \frac{8\pi hc}{\lambda^5} d\lambda \frac{1}{[\exp{(hc/\lambda k_B T)} - 1]}.$$

Check this!

(a) As the (linear) size of the universe increases by a factor of f, the wavelength increases in proportion to a new value: $\lambda' = f\lambda$.
Therefore the energy density du' in the new wavelength range λ' to $\lambda' + d\lambda'$ decreases for two different reasons: (i) The volume of the universe increases while the number of photons remains the same; and (ii) the energy of each photon is decreases because of the change in wavelength.
Show that

$$du' = f^{-a} du.$$

and determine a.

(b) Use the explicit expression for du given above on the right hand side of the preceding equation. Next rewrite it in terms of the new λ' to deduce that du' is identical to Planck's formula in terms of λ' at a new temperature

$$T' = f^{-b} T$$

and hence determine b.