

PHYSICS 846

Home Work Assignment # 1

1/13/2009

Due: Tues., Jan. 20, 2009

Completed assignments should be placed in grader's mail box in PRB by 5:00 PM

1. Define a *quasi-static process* as one in which the system is always close to an equilibrium configuration; hence its trajectory may be displayed on a state diagram, e.g., on a (p,V) diagram. A *reversible process* is one that can also occur in reverse.

(a) Is every quasi-static process also a reversible process?

(b) Is every reversible process quasi-static?

In both the questions above, if the answer is "yes" please explain why. If your answer is "no" give an example.

2. Thermodynamics can be very confusing if one relies only on "formal" manipulations of equations, without any thought. Below is an example of a plausible looking, but *incorrect*, line of mathematical reasoning that reaches a conclusion that is clearly wrong.

Step 1. (First Law) $dU = \delta Q - pdV$

Step 2. (definition of specific heat) $\delta Q = C_v dT$

Step 3. Thus: $dU = C_v dT - pdV$ from which we reach the manifestly *wrong* conclusion that

$$\left(\frac{\partial U}{\partial V}\right)_T = -p$$

(a) Give a simple counterexample that clearly shows that the last result is in general incorrect.

(b) Find the flaw in the reasoning given above.

3. Consider a system with an equation of state:

$$p = \alpha T^4$$

where α is a constant, and a specific heat at constant volume:

$$C_v = 12 \frac{pV}{T}$$

- (a) Calculate the internal energy $U(p, V)$. You may use the fact that the internal energy per unit volume is only a function of the temperature.
- (b) Calculate the curve in the (p, V) diagram followed by the system when it undergoes an adiabatic transformation.
- (c) Sketch a Carnot cycle of this system on the (p, V) diagram.

4. A rubber band of length L is subject to a constant force f at temperature T . Assume you are given the equation of state $f = f(L, T)$.

(a) If the rubber band is extended reversibly while keeping its temperature constant, calculate how much heat δQ flows into it if it is extended a small amount dL . *Hint*: First derive a Maxwell relation relevant to this problem. (Such relations follow from the equality of cross derivatives independent of the order of differentiation). Then use it to connect the derivative of interest to a derivative that only involve the state variables f, T, L .

(b) If $f(T, L)$ is a monotonically increasing function of temperature for any fixed L , will the rubber band expand or contract if heat is added to it? Compare this with the behavior you would expect from a metallic wire.

(c) In addition you are given the specific heat at constant length $C_L(T, L)$. Find the change in temperature of the rubber band when it is extended adiabatically (i.e., in thermal isolation) by a small amount dL .

(d) Show that a knowledge of $C_L(T, L)$ and $f(T, L)$ are sufficient to obtain the internal energy function $U(T, L)$ (up to an overall additive constant). In other words, show that both partial derivatives of U are completely determined by this information.

(e) For a given equation of state $f(T, L)$ can $C_L(T, L)$ be an arbitrary function? If not, describe the constraint on it.

(f) Take $f(L, T) = a(L)T$ as the equation of state of the rubber band for the questions that follow: (i) What constraint does this impose on the specific heat $C_L(T, L)$? (ii) Taking $a(L) = \alpha L$ and $C_L = \beta$ where both α and β are constants, find the entropy of the rubber band as a function of U and L .

5. It is postulated (Bekenstein-Hawking) that the entropy of a black hole is proportional to the area of its event horizon $S = \sigma A$. The proportionality constant $\sigma = k_B c^3 / 4hG$ depends only on fundamental constants, and $A = 4\pi R^2$, where the event horizon R is the distance at which the escape velocity of a body is equal to the speed of light. (Although non-relativistic, this condition turns out to give the exact answer for R).

(a) For a mass M black hole calculate the internal energy (using Einsteins relation between mass and energy) and entropy.

(b) When you throw something into a black hole and increase its mass by dM , it should be thought of as transferring heat to it without doing any work. Use thermodynamics to find how the temperature of the black hole depends on its radius. Estimate the temperature of a black hole of radius 1 km.

6. Think about Problem 12 of Chapter 1 in Kardar's book (reproduced below).

(a) The solar system originated from a dilute gas of particles, sufficiently separated from other such clouds to be regarded as an isolated system. Under the action of gravity the particles coalesced to form the sun and planets. The motion and organization of planets is much more ordered than the original dust cloud. Why does this not violate the second law of thermodynamics?

(b) The nuclear processes of the sun convert protons to heavier elements such as carbon. Does this further organization lead to a reduction in entropy?

(c) The evolution of life and intelligence requires even further levels of organization. How is this achieved on earth without violating the second law?

Read the article "Entropy and Evolution" by D. F. Styer, Am. J. Phys. **76**, 1031 (2008).