PHYSICS 633

Home Work Assignment # 2 04/03/2008

Due: Thursday, April 10, 2008

All problem numbers are from the text book by Griffiths.

• **Degenerate perturbation theory**: 1) Problem 6.9 (a), (b), (c), (d).

• **Two useful theorems**:

2) The *virial theorem* in 3 dimensions: Problem 4.40 (a),(b),(c).

3) The *Feynman-Hellmann theorem*: Problem 6.32 (a) and (b).
Hint for part (a): *Ignore* the (in my opinion) misleading hint in the book about using eq. (6.9), since this theorem is exact, and has nothing to do with first order perturbation theory. Instead proceed in part (a) as follows: Given that $\mathbf{H}(\lambda)|\psi_n(\lambda)\rangle = E_n(\lambda)|\psi_n(\lambda)\rangle$, compute $\frac{d}{d\lambda}\langle\psi_n(\lambda)|\mathbf{H}(\lambda)|\psi_n(\lambda)\rangle$ and use the normalization $\langle\psi_n(\lambda)|\psi_n(\lambda)\rangle = 1$.

• **Fine structure of the Hydrogen atom**:

4) Compute the expectation value $\left\langle \frac{1}{r} \right\rangle$ in Hydrogen atom states, using the virial theorem: Problem 6.12.

5) Compute $\left\langle \frac{1}{r} \right\rangle$ and $\left\langle \frac{1}{r^2} \right\rangle$ using Feynman-Hellmann: Problem 6.33 (a), (b).

5′) **(Optional):** This part of the problem is optional, but do read it! To compute $\left\langle \frac{1}{r^3} \right\rangle$ proceed as follows. Consider the radial momentum operator: $p_r = -i\hbar\,(\partial/\partial r + 1/r)$ and show that the radial part of the Hamiltonian (the left hand side) can be written as

$$\frac{-\hbar^2}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\right) = \frac{p_r^2}{2m}$$

Next use the fact that the expectation value in energy eigenstates of the Hydrogen atom: $\langle[H, p_r]\rangle = 0$. Explicitly evaluate the commutator and show that

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{\ell(\ell+1)a_0}\left\langle \frac{1}{r^2} \right\rangle.$$

Finally we can use the result of problem 6.33(b) to obtain the required result.

6) Putting together the results of (4), (5) and (5′) derive Eq. (6.66) for the Fine Structure of Hydrogen: Problem 6.17.