PHYSICS 827

Home Work Assignment # 6 11/5/2010

Due: Fri., Nov. 12, 2010 (by 5:00 PM in the grader's mail box).

The exercise numbers below are from Shankar's book (2nd edition).

1. Ex. 5.3.4 (page 167).

2. Ex. 5.4.2: only Part (a) (page 175)

3. Consider the one-dimensional potential barrier problem discussed in class: $V(x) = V_0 > 0$ for $0 < x < L$ and $V(x) = 0$ elsewhere. Give the details of the calculation (sketched in class) to derive the following results.
(a) Show that the transmission coefficient for a particle with energy $E > V_0$ is given by

$$T(E) = \frac{4E(E - V_0)}{4E(E - V_0) + V_0^2 \sin^2\left[\sqrt{2m(E - V_0)}\, L/\hbar\right]}.$$

Sketch the behavior as a function of Energy E, pointing out *scattering resonances.*
(b) For energy $E < V_0$, the transmission coefficient is

$$T(E) = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \sinh^2\left[\sqrt{2m(V_0 - E)}\, L/\hbar\right]}.$$

What is the L-dependence of the *tunneling probability* in the limit of a large barrier height?

4. We have solved all our one-dimensional QM problems in the coordinate representation. Sometimes it is useful to work in the **Momentum Representation**, which is introduced here and used in the following problems.
In this representation, we project states onto the basis of eigenstates $|p\rangle$ of the momentum operator $\widehat{P}$ using the notation $\langle p|\psi\rangle = \psi(p)$.
(a) What is the relation between $\psi(p)$ and the coordinate representation $\psi(x)$? (I follow the unfortunate, but standard, practice amongst physicists and use the same symbol to denote two different functions, with the argument indicating whether I am talking about the coordinate or momentum

representation).
(b) Show that the position operator

$$\widehat{X} \to i\hbar \frac{d}{dp}$$

in the momentum representation. (Hint: one way to proceed is to start with the commutation relation of $\widehat{X}$ and $\widehat{P}$.)
(c) Write the time-independent Schrodinger equation for the wavefunction $\psi(p)$ of a particle of mass m moving in one dimension in a potential $V(x)$ (which has a well defined Fourier transform).

5. Solve the problem of the bound state in the potential $V(x) = -V_0 a \delta(x)$ working in the momentum representation. Show that $\psi(p)$ is a Lorentzian and find the binding energy.
Compare your result with your earlier solution in the coordinate representation (Ex. 5.2.3). (The integral needed here is elementary and occurs very often in physics. It is simple to do using complex analysis, but in case you are not familiar with that, use Mathematica or a table of integrals.)

6. Consider a quantum particle of mass m subjected to a uniform force field: $V(x) = -Fx$; this could be a particle in a gravitational field or a charge in an electric field. The time-independent Schrodinger equation in the coordinate representation is a second order differential equation that cannot be solved in terms of elementary functions!
(a) Find the equation for $\psi_E(p)$, corresponding to an energy eigenvalue E, and show that it can be easily solved.
(b) Thus find an integral expression for $\psi_E(x)$. Check that this is related to the Airy function (about which you can learn from any standard reference on special functions or from that fountain of knowledge in the 21st century called Wikipedia).

7. Ex. 7.3.7 (page 202).