PHYSICS 827

Home Work Assignment # 1 09/24/2010

Due: Fri., Oct. 1, 2010

Completed assignments should be placed in the grader N. Ramalingam's mail box in PRB by 5:00 PM.

1. Find order-of-magnitude estimates of the deBroglie wavelength $\lambda = 2\pi\hbar/p$. Make sure you justify the use of relativistic or non-relativistic formulas.

(a) A small grain of dust (mass $\simeq 10^{-12}$g; speed $\simeq$ 1mm/s). Compare λ with the size of the particle $\simeq 1\mu$m.

(b) A thermal neutron (i.e, one whose kinetic energy is of order $k_B T$ with $T \simeq 300$K). What interesting physics problems could thermal neutrons be used to probe? What kind of EM radiation has a comparable wavelength?

(c) A 1 GeV electron. What could these electrons be used to probe?

2. Never underestimate the power of *dimensional analysis*!
Let us write any quantity A of interest to us in the form

$$A \sim M^\alpha L^\beta T^\gamma$$

where α, β, γ are integers. Thus, for example, an energy $E \sim ML^2/T^2$. Here M is a mass, L a length and T a time scale. We use $\sim$ rather than $=$ because we are omitting a *dimensionless* multiplicative constant on the RHS. This number will likely be of order unity, provided we have chosen characteristic scales M, L and T appropriate for the problem at hand.

Suppose we are interested in the quantum mechanics of a particle of mass M and a characteristic length scale L (this could, e.g., be the "size" of the system). The only other input we have – at this stage – is that *Planck's constant* $\hbar$, whose units are Joule-second, *must enter the final result.*

Using dimensional analysis alone determine the form of the following quantities in terms of M, L and $\hbar$:
(a) time, (b) momentum, (c) energy.

Now let us be more ambitious and try to learn as much as we can about the quantum mechanics of the hydrogen atom using dimensional analysis.

The potential energy of the electron of mass M moving around the (infinitely heavy) proton is of the form Q^2/L (Coulomb's law) where we need to introduce a new variable Q, the charge. Again, we drop all dimensionless factors (including the physically crucial negative sign which implies attraction). Using the results you obtained above, together with this P.E., find expressions for the
(i) characteristic size and (ii) the characteristic energy scale of the hydrogen atom.
Compare your dimensional analysis answers with the well known results for the Bohr radius and binding energy of the hydrogen atom.

3. Consider the set of all complex polynomials of degree $\leq N$:

$$A(z) = A_0 + A_1 z + A_2 z^2 + \ldots + A_N z^N,$$

where z is a complex number. Using "natural" definitions of "vector addition" and "scalar multiplication", show that this set is a complex vector space. Give a basis set for this space, and find its dimension.

4. Consider the set of all real periodic functions $f(x)$ for $0 \leq x \leq 2\pi$. Show that this is a real vector space. Write down a suitable set of linearly independent functions. What is the dimension of this vector space?

5. Shankar, Ex. 1.3.4.

6. Shankar, Ex. 1.4.1.