<u>PHYSICS 880.06</u>

Home Work Assignment # 4 10/18/2011

<u>Due:</u> Tue., Oct. 25, 2011.

Term Paper:
Please note that you need to finalize your term paper topic and give me an outline and list of references by October 25.

Problem 1: *Spin susceptibility of the Free electron gas:*

Consider the Zeeman effect of an external magnetic field H on the free, non-interacting electron gas in 3D with dispersion $\epsilon(\mathbf{k}) = \hbar^2 k^2/2m$. We ignore here the orbital effect of the external field. We thus focus only on the *spin susceptibility* of electrons, which is in fact possible to measure using NMR Knight shift experiments.

In the absence of the field, the Hamiltonian is $\mathcal{H}_0 = \sum_{\mathbf{k},\sigma} \epsilon(\mathbf{k}) n_\sigma(\mathbf{k})$, where the occupation $n_\sigma(\mathbf{k}) = 0$ or 1 depending on whether the one-electron state $|\mathbf{k}, \sigma\rangle$ is empty or filled. The s_z label $\sigma = \uparrow, \downarrow$. Upon turning on the field, the Hamiltonian changes to

$$\mathcal{H} = \sum_{\mathbf{k}} [\epsilon(\mathbf{k}) - \mu_B H] n_\uparrow(\mathbf{k}) + \sum_{\mathbf{k}} [\epsilon(\mathbf{k}) + \mu_B H] n_\downarrow(\mathbf{k})$$

Here each electron has a magnetic moment $g\mu_B S = \mu_B$, since $g = 2$ and $S = 1/2$. The Bohr magneton $\mu_B = e\hbar/2mc \simeq 0.58 \times 10^{-8} eV/Gauss$.

(a) Carefully plot the two densities of states $g_\sigma(\epsilon)$ for $\sigma = \uparrow, \downarrow$ as functions of energy ϵ. Following standard convention here, plot ϵ along the y-axis, and the up/down DOS along the positive/negative x-axis.

(b) At $T = 0$ we fill up the states up to the chemical potential $\mu_\uparrow = \mu_\downarrow = \epsilon_F$. Why are the two chemical potentials equal?

(c) Calculate the densities of up and down electrons, $n_\uparrow$ and $n_\downarrow$, at $T = 0$ by integrating the occupancy times the DOS. (Be sure to pay attention to the limits of integration).

(d) Find $n_\uparrow$ and $n_\downarrow$ to linear order in H by making a low field approximation $\mu_B H \ll \epsilon_F$. Comment on the validity of this approximation for magnetic fields available in the laboratory.

(e) Compute the magnetization (per unit volume) of the sample $M = \mu_B(n_\uparrow - n_\downarrow)$, and hence the susceptibility $\chi = \partial M/\partial H$. Show that

$$\chi = \mu_b^2 g(\epsilon_F)$$

Argue that the $T = 0$ calculation is in fact valid for all $T \ll \epsilon_F$ and gives, in principle, an experimental way to estimate the density of states at the chemical potential $g(\epsilon_F)$ (provided the assumptions of the free, non-interacting gas are valid!).

(f) Remind yourself of the derivation of the Curie susceptibility for non-interacting "classical" magnetic dipoles:

$$\chi_{\text{classical}} = n\frac{\mu_b^2}{T}$$

where T is the temperature and n the density of dipoles with magnetic moment μ_B and I set $k_B = 1$. You don't need to show the derivation, but review it in your favorite statistical mechanics text book.

Use the classical result to get a simple physical picture of low temperature result of part (e). Recall that the Curie susceptibility is obtained if *all* the dipoles can align themselves along the magnetic field. Argue that in the quantum, degenerate regime ($T \ll \epsilon_F$) of interest, Pauli exclusion does not allow most spins to flip in an external field. Estimate the small fraction of the spins that can respond to the Zeeman field, and use this "effective number density" in Curie result to understand the T-independent result of part (e).