PHYSICS 880.06

Home Work Assignment # 3 10/11/2011

Due: Tue., Oct. 18, 2011.

1. *Free and independent electron gas in 2D*:
A & M (Ashcroft and Mermin) Chapter 2, Problem 1.

2. *Classical limit of Fermi-Dirac statistics*
A & M, Ch. 2, Problem 3 (a), (b), (c).
At what temperatures would Maxwell-Boltzmann statistics be applicable for conduction electrons in a metal? Is this realistic?
For what what materials could you use M-B statistics at room temperature?

3. Consider N *non-interacting* fermions of mass m in an external potential $U(\mathbf{r})$ described by the Hamiltonian:

$$H = \sum_{j=1}^{N} H_j \quad \text{where} \quad H_j = -\frac{\hbar^2}{2m}\nabla_j^2 + U(\mathbf{r}_j).$$

Ignore spin for the moment, i.e., consider spinless fermions; we will come back to spin in the last part of the question.

(a) If the one-particle Hamiltonian has the eigenfunctions

$$H_j \phi_n(\mathbf{r}_j) = \epsilon_n \phi_n(\mathbf{r}_j)$$

then show that the product wavefunction

$$\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N) = \prod_{j=1}^{N} \phi_j(\mathbf{r}_j)$$

is an eigenstate of the N-particle Hamiltonian and find its energy eigenvalue.

(b) The problem with Ψ_0 is that it does not satisfy the Pauli exclusion principle. (Why?) Consider the N-particle wavefunction, conveniently written as an $N \times N$ Slater determinant

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N) = \begin{vmatrix} \phi_1(\mathbf{r}_1) & \phi_1(\mathbf{r}_2) & \ldots & \phi_1(\mathbf{r}_N) \\ \phi_2(\mathbf{r}_1) & \phi_2(\mathbf{r}_2) & \ldots & \phi_2(\mathbf{r}_N) \\ \ldots & \ldots & \ldots & \ldots \\ \phi_N(\mathbf{r}_1) & \ldots & \ldots & \phi_N(\mathbf{r}_N) \end{vmatrix}$$

Show that $\Psi(\mathbf{r}_1, \ldots \mathbf{r}_N)$ satisfies Pauli exclusion and is an eigenfunction of H. Find its energy eigenvalue.

(c) Finally consider the non-interacting electron gas, where the fermions have spin-1/2. Consider N electrons, half of which have spin up and the other half spin down, in a box of size L^3, inside which the external potential $U(\mathbf{r})$ vanishes.

Generalize the results of part (b) and write down the ground state wavefunction of the N-particle system in terms of Slater determinants. Where does the Fermi wavevector k_F appear in your solution?