Approaching the Landscape: Computability of IIB/F-theory Instantons and Insights from MSSM-containing Quivers

Jim Halverson

November 5, 2010

Based on:
[1009.5386] with M. Cvetič and I. García-Etxebarria
[1006.3341] with M. Cvetič and P. Langacker
recent work with M. Cvetič and P. Langacker, *to appear*

Approaching the Landscape: Motivation

String vacua allow for many physical effects seen in our world:

- gauge symmetry, chiral matter, quantum gravity, inflation, etc.

In a perfect world . . .

- write down all vacua
- calculate all low-energy physical effects
- compare to our world

But there are problems . . . (obviously)

- landscape is enormous
- what is / specifies a vacuum?
- lack of perfect computational techniques.
- conjecture: there exists no algorithm which calculates all non-perturbative effects across the landscape. [MC, JH, IGE]

What Makes a "Good Approach"?

"Good approach" to study the landscape depends on:

- physics of interest.
  - $\rightsquigarrow$ a given physical effect usually depends on some subset of vacuum data.
  - $\rightsquigarrow$ coarse-grained approach w/ subsets $\Rightarrow$ non-trivial insights on broader patches of the landscape.
- issues of computability.
  - Can we actually calculate the physics we'd like to study?
  - $\rightsquigarrow$ "patchy" landscape, algorithmically, due to relation to number theory and the corresponding computability conjecture.

will give two major examples.

Outline

Type IIB Orientifolds

good overview for CY_3: [Blumenhagen, Braun, Grimm, Weigand]

Specify Basic Geometric and Brane Data:

- a Calabi-Yau threefold $\mathcal{X}$.
  - $\rightsquigarrow$ triple intersection numbers of divisors (are wrapped by $D7$'s)
- a holomorphic involution σ.
  - $\rightsquigarrow$ $O7/O3$-planes at fixed locus of σ.
- tadpole cancellation $\longrightarrow$ must introduce $D7$-branes.
  - $\rightsquigarrow$ condition on homology.

Relatively straightforward

$\rightsquigarrow$ when $\mathcal{X}$ is a complete-intersection in a toric variety!

D-Instanton Basics

[Blumenhagen, Cvetič, Weigand], [Ibañez,Uranga],[Florea, Kachru, McGreevy, Saulina]

in type IIB, consider **ED3 instantons**:

- pointlike in spacetime.
- wrap a divisor (four-cycle) in the Calabi-Yau.

Uncharged Modes: Instanton-Instanton Strings

$\rightsquigarrow$ counted by $\mathbb{Z}_2$-equivariant sheaf cohomology $h^i_\pm(D,\mathcal{O}_D)$:

Crucial: W contribution $\Leftrightarrow h^0_+(D,\mathcal{O}_D)=1$ (θ mode), rest zero.

Charged Modes: Instanton-Gauge Brane Strings

$\rightsquigarrow$ counted by sheaf cohomology $h^i(\mathcal{C},\mathcal{L}\otimes K_{\mathcal{C}}^{1/2})$:
$\mathcal{L}$ encodes flux on $D7$.

Geometric Indices: Necessary Constraints for W Contribution

Toric Geometry: The Basics

Very roughly: A generalization of weighted projective spaces.

$$\rightsquigarrow \quad (x_1, \ldots, x_r) \;\sim\; (\lambda^{Q_1^a} x_1, \ldots, \lambda^{Q_r^a} x_r)$$

Illustrative Examples:

$\mathbb{P}^2$

$$Q_1 : (x_1, x_2, x_3) \sim (\lambda^1 x_1, \lambda^1 x_2, \lambda^1 x_3)$$

$$SRI = \langle x_1 x_2 x_3 \rangle.$$

Coords	Q^1
x_1	1
x_2	1
x_3	1

dP_1

$$Q_1 : (x_1, x_2, x_3, x_4) \sim (\lambda x_1, \lambda x_2, \lambda x_3, x_4)$$

$$Q_2 : (x_1, x_2, x_3, x_4) \sim (x_1, \mu x_2, x_3, \mu x_4)$$

$$SRI = \langle x_1 x_3, x_2 x_4 \rangle.$$

Coords	Q^1	Q^2
x_1	1	0
x_2	1	1
x_3	1	0
x_4	0	1

The point: Can realize Calabi-Yau's as hypersurfaces or complete intersections in an ambient toric variety. $\rightsquigarrow$(e.g. the quintic)

Calculation: Indices and Line Bundle Cohomology

toric variety $\mathcal{A}$, CY_3 $\mathcal{X}$, instanton wraps divisor D. $\mathcal{A} \supset \mathcal{X} \supset D$.

Index Calculation: Integrating characteristic classes

- knowing Chern classes on $\mathcal{A}$, can calculate classes on D.
- integrate classes on D using intersection form.

$\rightsquigarrow$ diophantine equations in d_i of $D = \sum_i d_i D_i$.

Cohomology Calculation: Sheaf cohomology on D

- relate coh. on D to coh. on $\mathcal{A}$ by Koszul sequences.
- calculate relevant line bundle cohomology on $\mathcal{A}$.
  - $\rightsquigarrow$ ex. w/ conventional Čech complex.
    [Cvetič, García-Etxebarria, J.H.] (March)
  - $\rightsquigarrow$ efficient new theorem using intuition from SRI.
    [Blumenhagen, Jurke, Rahn, Roschy] (March), [Jow], [Rahn, Roschy]

$\mathbb{Z}_2$-equivariant Cohomology of Line Bundles

necessary and sufficient for W contribution.

Want to determine:

$\rightsquigarrow$ are modes orientifold odd or even?

Two key facts:

- New theorem tells you which sections contribute to cohomology.
- There is a natural σ-action on sections.

$\mathbb{Z}_2$-equivariant Cohomology Conjecture:

[Cvetič, García-Etxebarria, J.H.] $\sigma : x_i \mapsto -x_i$ for some i's.

determined by action on sections which contribute to cohomology.

Non-trivial Checks:

- sensible from σ-action on the Čech complex.
- checked in for thousands of bundles against Lefschetz genus.

Quick $\mathbb{Z}_2$-equivariant Example:

consider $\mathcal{O}_{\mathbb{P}^1}(2)$, with $\sigma : x_0 \mapsto -x_0$ as the involution.

$h^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(2)) = 3$ since $\mathcal{O}_{\mathbb{P}^1}(2)$ has global sections x_0^2, $x_0 x_1$, x_1^2.

then:

$$\sigma : x_0^2 \mapsto x_0^2$$

$$\sigma : x_0 x_1 \mapsto -x_0 x_1$$

$$\sigma : x_1^2 \mapsto x_1^2$$

$\rightsquigarrow \quad h^0_+(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(2)) = 2 \quad$ and $\quad h^0_-(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(2)) = 1$

Many applications and same conjecture made in [Blumenhagen, Jurke, Rahn, Roschy]

Computability of All Instanton Effects?

Computability Punchline:

can determine all instanton corrections

$$\Leftrightarrow$$

can find all solutions to a system of diophantine equations

Natural question: Is there an algorithm which solves them for some non-trivial patch of vacua?

- $\rightsquigarrow$ doable when $h^{11}_{\mathcal{X}} = 1$ (e.g., $\mathcal{X}$ = quintic)
- $\rightsquigarrow$ but maybe there's a more interesting patch?

"Factorizing" Manifolds: [Cvetič, García Etxebarria, J.H.]

Exact Non-perturbative Uncharged Superpotentials

If IIB compactification has:

- $\mathcal{X}$ w/ "factorizing" intersection form, $I_{\mathcal{X}} = D f_2$ for some D.
- no $D7$-branes with component along D.

Then:

- can solve the diophantine system.
  - $\rightsquigarrow$ can find all uncharged instanton corrections.

Remarks:

- includes *many* elliptically fibered threefolds.
- factorization structure greatly elucidated in $\mathcal{X} \subset \mathcal{A}$ cases.

A Concrete Example: Elliptic fibration over dP_2

Manifold, defined: the toric space $\mathcal{A}$ and CY $\mathcal{X} \subset \mathcal{A}$ are given by

	x_1	x_2	x_3	x_4	x_5	x	y	z	
$\mathbb{C}^*$	**1**	**1**	**1**	**0**	**0**	6	9	0	M
$\mathbb{C}^*$	**0**	**1**	**0**	**0**	**1**	4	6	0	N
$\mathbb{C}^*$	**1**	**0**	**0**	**1**	**0**	4	6	0	O
$\mathbb{C}^*$	0	0	0	0	0	**2**	**3**	**1**	P

$$SRI = \langle x_1x_3, x_1x_4, x_2x_3, x_2x_5, x_4x_5, xyz \rangle$$

$\mathcal{X} = \{y^2 = x^3 + f(x_i)xz^4 + g(x_i)z^6\}$ $\Leftarrow$ Weierstrass Equation

$I_{\mathcal{X}} = P(MN + MO - MP - NP - OP - M^2 - N^2 - O^2 + 7P^2)$

Comment 1: note that $[D_8] = P$ factorizes out. $D_8 = dP_2$ base!
Comment 2: manifold used for semi-realistic GUT model.

Orientifold Involution: $\sigma : x_2 \mapsto -x_2$

Now consider: ED3 instanton on $D = mM + nN + oO + pP$ with $m, n, o, p \in \mathbb{Z}$.

Uncharged Modes: Necessary Constraint

$$\chi(D, \mathcal{O}_D) = -\frac{1}{6}(3m^2p - 6mnp + 3n^2p - 6mop + 3o^2p + 3mp^2 + 3np^2 + 3op^2 - 7p^3 - 6m - 6n - 6o + p) \stackrel{?}{=} 1.$$

Absence of Charged Modes for Given $D7$, either:

- D doesn't intersect D_{D7}. (sufficient)
- $D \cap D_{D7} = \mathcal{C} = \mathbb{P}^1$. (necessary if $D \cap D_{D7} \neq \emptyset$, not sufficient)

$\rightsquigarrow$ in short, because $h^i(\mathbb{P}_1, \mathcal{O}(-1)) = (0, 0)$. Not uncommon.

Non-intersection conditions:

$$D_2 \cdot D = 0 \qquad \leftrightarrow \qquad p = 0 \qquad o = 2p$$
$$D_3 \cdot D = 0 \qquad \leftrightarrow \qquad p = 0 \qquad m = n + o$$
$$D_5 \cdot D = 0 \qquad \leftrightarrow \qquad p = 0 \qquad m = n.$$

Intersect at $\mathbb{P}^1$ conditions:

$$\chi(D \cdot D_2) \; = \; -2p(o-p) \; = \; 2 - 2g \overset{\mathbb{P}^1}{=} 2$$
$$\chi(D \cdot D_3) \; = \; p(2m - 2n - 2o + p + 1) \; = \; 2 - 2g = 2$$
$$\chi(D \cdot D_5) \; = \; -p(2m - 2n - p - 1) \; = \; 2 - 2g = 2$$

Solution: Restricts to **finite set** of contributing instantons.

Then $\chi(D, \mathcal{O}_D) = 1 \Rightarrow$ only $D = P$ contributes to the uncharged superpotential.

This solution algorithm applies **generically** to compactifications satifying the above conditions.

- $\rightsquigarrow$ demonstrates "patchiness" of the landscape.
- $\rightsquigarrow$ solves an interesting physical problem.

Other physical problems?

Study Chiral Matter Spectra:

- IIB, depends on topological data of $\mathcal{X}$ and flux.

- but Calabi-Yau manifolds are fairly detailed . . . gets tedious
- can we **specify less**, which **gains breadth**, and still **learn interesting things**?

String Theoretic Constraints on Chiral Matter

to have tadpole cancellation and a massless $U(1)_Y$:
constraints on the homology of branes must be satisfied.

gives necessary constraints on chiral spectrum

- some are standard anomaly cancellation constraints.
  - $\rightsquigarrow$ beautiful geometric picture of non-abelian cancellation.
- also some genuinely stringy constraints.
- holds across weakly coupled type II, RCFT, (maybe more?)
  [Cvetič, J.H., Richter]
  [Anastasopoulos, Leontaris, Richter, Schellekens]

Key Point:

- use subset of vacuum data to learn as much as possible.
- rule out many D-brane quivers without specifying geometry.

Questions:

- Which matter fields beyond the MSSM (in given scenario) are preferred by chiral matter constraints (if any)?
- Which "preferences" are already present in field theory? Which are stringy (if any)?

Scenario: Three-stack quivers (first step, for simplicity)

- realize on D-brane stacks a,b,c with $U(3)_a \times U(2)_b \times U(1)_c$
- hypercharge realized only as $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c$

  or $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$
- First embedding is "Madrid embedding".

Strategy / Approach:

[Cvetič, J.H., Langacker], to appear

- **write down** all three-stack quivers with exact MSSM.
  - $\rightsquigarrow$ even quivers which don't cancel tadpoles.
- **write down** all tadpole-cancelling matter additions (w/ n_{max}).
- **determine** which matter additions leave hypercharge massless.
  - $\rightsquigarrow$ prefer some types of matter over others?

Three-stack Madrid Embedding: Facts

- tadpoles take form $tad_{a,b,c} = (0, \pm 2n, 0)$ for $n \in \{0, \ldots, 7\}$.
- all exact MSSM quivers satisfy masslessness constraints.

Preliminary Results: Madrid Embedding

"Preferred" Matter Additions? (Tinker Toys)

- MSSM singlets $\tiny\yng(1,1)_b$, hypercharge-less SU(2) triplets $\tiny\yng(2)_b$, and extra chiral Higgs pairs, e.g. $(\bar{b}, c) + (\bar{b}, \bar{c})$.

$\rightsquigarrow$ don't contribute to masslessness condition, use as building blocks to cancel tadpoles.

$\rightsquigarrow$ e.g., could add chiral Higgs pair + singlet to cancel $tad_b = 4$ $\Rightarrow S\widetilde{H}_u\widetilde{H}_d$ type coupling.

Matter Bounds:

- e.g. if just adding n_S chiral MSSM singlets, have $n_S \leq 7$.
- constraints on matter not present in field theory.

Singlet-Extended MSSM Quivers

[Cvetič, Halverson, Langacker] **June 2010**

Systematic Analysis of Three-Stack and Four-Stack Quivers.
(along lines of [Cvetič, J.H., Richter](3), [Cvetič, J.H., Langacker, Richter])

Key addition: Add a singlet S with coupling SH_uH_d.

Constraints

- MSSM + S or MSSM + $3N_R + S$.
- necessary for tadpole cancellation and a massless $U(1)_Y$.
- many phenomenological constraints.
- H_uH_d pert. and non-pert. forbidden, but SH_uH_d present.
- LH_uS is absent, so $S \neq N_R$.
- can generate S^n term without pheno. drawbacks.

Singlet-Extended MSSM Quivers

Results

- Only three quivers survive all constraints.
  - Singlets realized as bifundamental of $U(1)$'s, rather than $\begin{smallmatrix}\square\\\square\end{smallmatrix}_b$.
- "Model-independent" quivers:
  - no S^n superpotential term is phenomenologically fixed.

$\rightsquigarrow$ can in principle have $W \supset f(S)$, f a polynomial.

$\rightsquigarrow$ all three quivers could in principle realize the nMSSM, S^2 model, or NMSSM. Depends heavily on details of global embedding.

Summary

"Good approach" to study the landscape depends on:

- physics of interest.
  - $\rightsquigarrow$ coarse-grained approach w/ subsets of vacuum data $\Rightarrow$ non-trivial insights on broader patches of the landscape.
- issues of computability.
  - $\rightsquigarrow$ "patchy", algorithmically, due to relation to number theory and the corresponding computability conjecture.

Gave two major examples.

Summary

Example One: All Instanton Corrections to Uncharged W

- **new conjecture:** for certain $\mathbb{Z}_2$-equivariant cohomology.

Solvable Patch: "Factorizing" Calabi-Yau Threefolds in IIB

- manifolds with a special divisor that participates in every non-zero triple intersection. i.e. $I_{\mathcal{X}} = D f_2$ for some D.
- includes many elliptically fibered threefolds.
- **Key point:** certain diophantine equations become linear $\Rightarrow$ can solve for all instantons contributing to uncharged W.

Summary

Example Two: Semi-realistic Quivers in type II

- "coarse-grained" approach
- **Stringy Constraints on Chiral Matter**
  - many quiver/matter additions might satisfy tadpole constraints, but which leave $U(1)_Y$ massless?

  Three-stack Madrid Embedding:
  - MSSM singlets, hyperchargeless $SU(2)$ triplets, chiral Higgs pairs are **favored additions**.
  - string-theoretic "matter bounds", e.g. $n_S \leq 7$.

Constraints necessary for tadpole cancellation:

$$\#(a) - \#(\overline{a}) + (N_a - 4)\#\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}_a\right) + (N_a + 4)\#(\square\square_a) = 0$$

Constraints necessary for massless $U(1)_Y$:

$$q_a\, N_a \left(\#(\square\square_a) + \#\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}_a\right)\right) = \sum_{x \neq a} q_x\, N_x \left(\#(a, \overline{x}) - \#(a, x)\right)$$