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Computability of non-perturbative effects in the string theory landscape

- IIB/F-theory perspective -

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Based on [1009.5386] with M. Cvetič and J. Halverson.



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Phenomenology from the top-down

String theory has a very large number of consistent vacua (perhaps infinite).

- How do we find our vacuum in this large class?
- Is there anything in the dynamics of string theory that makes our vacuum special?

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Phenomenology from the top-down

Any attempt to answer these questions in detail presumably requires knowledge of the low energy dynamics in each vacuum:

- What are the light fields?
- What is their effective action?

Many backgrounds in which these questions can be (partially) answered are known (IIA intersecting branes, IIB large volume, F-theory, M-theory, branes at singularities, heterotic, free fermions, ...). Presumably, many more are there but are still unknown.

A first step in approaching this question is one of *computability*.

What can we compute, even if it is just in principle?

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Defining "compute"

We want to understand which quantities are computable given:

- A classical algorithm (something implementable in a deterministic Turing machine).
- Arbitrarily large, but finite, computing space/time.

These assumptions are too optimistic, but still do not seem enough to tame the landscape: given enough complexity in the landscape, some questions *cannot* be answered in such a way. We focus on non-perturbative effects in large volume IIB compactifications.

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Large volume IIB backgrounds

Our motivation is studying computability, so we need configurations where we have as much control as possible (without being trivial): **complete intersections in ambient toric spaces**.

In order to construct Calabi-Yau spaces we take hypersurfaces in \mathcal{A} , defined by some equation (or system of equations) $f(x_i) = 0$ of appropriate degree.

Generality

The Calabi-Yau threefolds obtained by taking a hypersurface $f(x_i) = 0$ in a 4d ambient space have been classified by Kreuzer and Skarke. There are at least $\approx 3 \cdot 10^4$ of these (at most $\approx 5 \cdot 10^8$). The generic class is expected to be much larger.

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Large volume $\mathcal{N} = 1$ compactifications

In order to construct a semi-realistic background we need to add an open string sector. We introduce $O7^-$ planes by quotienting A by involutions of the form:

$$x_i \mapsto \pm x_i \tag{1}$$

This gives rise to a fixed point locus in X given by the intersection of f = 0 and $x_1 = 0$ in A.

D7 branes can then be introduced by specifying which divisors they wrap, making sure that there are no anomalies.

Easily lifted to F-theory: [Collinucci (2); Blumenhagen, Grimm, Jurke, Weigand; Blumenhagen, Collinucci, Jurke; Cvetič, I.G.E., Halverson]

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D-brane instantons

In this class of compactifications, the relevant non-perturbative F-terms come from euclidean D3 branes. That is, D3 branes wrapping a (holomorphic) 4-cycle in X, and point-like in spacetime.

We restrict ourselves to instantons which do not admit a gauge theory description.



The most important property of a D-brane instanton is its spectrum of *zero modes*:

- Neutral zero modes: coming from ED3-ED3 strings.
- Charged zero modes: coming from ED3-D7 strings.

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D-brane instantons

Neutral zero modes

Consider an instanton wrapping a divisor D mapped to itself (not pointwise) under the $O7^-$ orientifold action. The gauge group on its worldvolume is $O(1) = \mathbb{Z}_2$.

Its spectrum of zero modes is given by [Blumenhagen, Cvetič, Kachru, Weigand; Blumenhagen, Collinucci, Jurke]:

Zero modes	Number
$(X_{\mu}, \theta^{\alpha})$	$h^0_+(D,\mathcal{O}_D)=1$
$\overline{ au}_{\dot{lpha}}$	$h^0(D,\mathcal{O}_D)=0$
γ_{lpha}	$h^1_+(D, \mathcal{O}_D)$
$(\omega, \overline{\gamma}_{\dot{lpha}})$	$h^1(D,\mathcal{O}_D)$
χ_{lpha}	$h^2_+(D, \mathcal{O}_D)$
$(c, \overline{\chi}_{\dot{lpha}})$	$h^2(D, \mathcal{O}_D)$

(Generically fermions are the most important modes.)

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D-brane instantons

Necessary conditions for contributing to the superpotential

Superpotential contributions are particularly interesting in phenomenological applications. An instanton contributing to the superpotential has exactly two neutral zero modes θ^{α} . I.e., we need:

$$h^i_{\pm}(D,\mathcal{O}_D) = 0 \tag{2}$$

for i > 0, and also:

$$h_{+}^{0} = 1. (3)$$

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Necessary conditions for contributing to the superpotential This gives a couple of useful conditions. Notice that for an instanton contributing to the superpotential, since only $h^0_+(D, \mathcal{O}_D) = 1$ is non-vanishing:

$$\sum_{i=0}^{2} (-1)^{i} (h_{+}^{i}(D, \mathcal{O}_{D}) \pm h_{-}^{i}(D, \mathcal{O}_{D})) = 1$$
(4)

These sums are actually indices, and can be computed topologically. For the + sign (GRR):

$$\sum_{i=0}^{2} (-1)^{i} (h_{+}^{i}(D, \mathcal{O}_{D}) + h_{-}^{i}(D, \mathcal{O}_{D})) = \int_{D} \mathrm{Td}(TD)$$
 (5)

and for the minus sign (Lefschetz):

$$\sum_{i=0}^{2} (-1)^{i} (h^{i}_{+}(D, \mathcal{O}_{D}) - h^{i}_{-}(D, \mathcal{O}_{D})) = \int_{M^{\sigma}} \frac{\mathrm{Td}(TM^{\sigma})}{\mathrm{ch}_{\sigma}(\wedge_{-1}\overline{NM^{\sigma}})} \quad (6)$$

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Necessary conditions for contributing to the superpotential

Writing the divisor D in terms of a basis of divisors D_i : $D = \sum d_i D_i$, we have that the index formulas give rise to diophantine equations. For example:

$$1 = \int_D \operatorname{Td}(TD) = a_{ijk} d_i d_j d_k + b_{ij} d_i d_j + c_i d_i$$
(7)

with a, b, c depending on the particular geometry we choose.

This is a first hint of the connection between instanton dynamics and number theory/computability.

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General discussion

In general we are interested in computing things other than superpotentials, i.e. generic F-terms, so we need to compute all $h^{\bullet}_{\pm}(D, \mathcal{O}_D)$.

In order to do this, we can use the Koszul resolution, relating cohomology of X to cohomology of A:

$$0 \to H^{0}(\mathcal{A}, \mathcal{O}_{\mathcal{A}}(D-X)) \to H^{0}(\mathcal{A}, \mathcal{O}_{\mathcal{A}}(D)) \to H^{0}(X, \mathcal{O}_{X}(D)) \to$$
$$\to H^{1}(\mathcal{A}, \mathcal{O}_{\mathcal{A}}(D-X)) \to H^{1}(\mathcal{A}, \mathcal{O}_{\mathcal{A}}(D)) \to H^{1}(X, \mathcal{O}_{X}(D)) \to \dots$$
(8)

For toric varieties we know how to compute line bundle cohomology systematically [Cox,Little,Schenck], [Cvetič, I.G.-E., Halverson] (and efficiently [Blumenhagen, Jurke, Rahn, Roschy; Jow; Rahn, Roschy]).

Equivariant version: [Cvetič, I.G.-E., Halverson; Blumenhagen, Jurke, Rahn, Roschy]

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Computability and diophantine equations

The simplest question that we can pose about a diophantine equation is: "Does it have a solution?". (In our context: "is there some instanton that satisfies the given necessary condition on its zero modes?")

Hilbert's 10th problem

Construct an algorithm that, given an arbitrary diophantine equation, determines in finite time whether it has a solution.

Physically, if we had a way of computing non-perturbative F-terms for an arbitrary compactification, we would have a way of determining if a large class of diophantine equations have solutions.

If the landscape is rich enough, we are effectively solving Hilbert's 10th problem.

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Computability and diophantine equations

Matiyasevich's solution to Hilbert's 10th problem

No such algorithm can exist.

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Computability and diophantine equations

This strongly suggests that there is no systematic algorithm for computing non-perturbative F-terms in a generic string theory vacuum.

Still, we need to be more precise: the index formulas are only necessary conditions, generically we are interested in asking questions that require knowledge of the exact spectrum of zero modes.

We show that the set of string vacua for which a given non-perturbative effect is present is recursively enumerable inside the set of all string vacua.

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Recursively enumerable sets

Definition

A subset $S_0 \subset S$ is *recursively enumerable* if there exists some algorithm that, given $x \in S$, halts iff $x \in S_0$.

(Roughly: recursively enumerable \approx computable, but may take infinite time to find the answer.)

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Recursive sets

Definition

A subset $S_0 \subset S$ is *recursive* (or *computable*) if there exists some algorithm that, given $x \in S$, determines in finite time whether $x \in S_0$.

We conjecture that $S_{\mathcal{P}}$ is *not* recursive. I.e., there is no way of computing non-perturbative F-terms systematically across the whole landscape.

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The string theory landscape

Evidence for the conjecture

Theorem (Matiyasevich)

A subset is recursively enumerable iff it is diophantine.

So we can view the problem of computability in the string theory landscape as an specialization of Hilbert's 10th problem to the set of diophantine equations arising in string theory.

The larger the landscape, and the larger the set of properties we want to compute, the more likely that the negative solution of Hilbert's 10th problem applies to us.

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The string theory landscape

Related results

- Turing's resolution of the halting problem.
- Some problems involving dynamics in the landscape are known to be NP-complete [Denef,Douglas].
- Simple statements about toy models of dynamics in the landscape are unprovable in ZFC [Williamson].

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Conclusions

- There is a very interesting computability structure behind non-perturbative effects in string theory, also very related to number theory.
- It suggests that there is no algorithmic way of computing non-perturbative F-terms across the whole landscape.
- If the conjecture is true, the landscape is inherently "patchy" (and never fully calculable).
- The viewpoint seems to be useful also for applications: elliptic fibrations. [See Jim's talk.]

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Physical implications

How do we compute in the landscape?

For practical calculations, we do not need the exact F-terms, so we can just truncate the instanton numbers.

- Can we do better in general than straightforward scans?
- How to describe the dynamics of string theory?

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Open questions

- Extend and generalize the discussion to other corners of the landscape.
- Use more powerful techniques in number theory for analyzing the resulting diophantine equations (elliptic curves, for example).
- Prove non-computability (case by case...).
- Formulate a computational model suitable for the physics on the landscape (non-deterministic Turing machine?).