PHYSICS 730 Summer 2005 August 2 midterm

Name:_____

Each problem is worth 20 points. Clearly show the steps in each calculation.

1) A capacitor C, inductor L, and resistor R are connected in series and attached to a battery with emf $\mathcal{E}$. The charge on the capacitor at $t = 0$ is zero, and which point the emf is turned on so that $\mathcal{E} = \mathcal{E}_0\theta(t)\theta(\tau - t)$. In other words it is turned on at $t = 0$ and off at $t = \tau$. Assume $R/2L > 1/\sqrt{LC}$. Find the charge on the capacitor, $q(t)$. The equation governing this circuit is:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$$

2) Determine the Fourier series of the rectified sine wave

$$f(t) = |sin(\omega t)|, \qquad -\pi/\omega \leq t < \pi/\omega.$$

3) Use the Laplace transform to solve

$$y''(t) + 3y'(t) + 2y(t) = e^t, \quad y(0) = 1,\ y'(0) = 0.$$

4) A charge Q is uniformly distributed on a loop in the $x-y$ plane defined by the equation $2x^2+y^2 = 1$. Using delta functions, give the charge distribution.

5) The LRC circuit from problem 1 is driven by delta function pulses $\mathcal{E} = \mathcal{E}_0\delta(t)$ that are repeated with period τ (*i.e.*, at ..., $-2\tau, -\tau, 0, \tau, 2\tau, ...$). Find $q(t)$ with the same assumptions as in problem 1.