PHYSICS 633  Spring 2007  Exam #1 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer.

1) \( V(x) = \delta x^4 \). Use the trial state \( \psi(x) = e^{-ax^2} \) to find an upper bound on the ground state energy. Plot \( a \) as a function of \( \delta \) and discuss why it is reasonable, e.g. why it behaves as it does for very small \( \delta \) and very large \( \delta \).

2) Two identical spin-1/2 fermions are confined to a 2-dimensional box, 0 < \( x < a \) and 0 < \( y < a \). (a) Compute the energies of the ground state and first excited state and their degeneracies (i.e., how many different states have that energy). (b) A “small” perturbative potential \( V(r_1, r_2) = \alpha \delta^2 (r_1 - r_2) \) is added to the potential that confines these fermions to a box. Use first-order perturbation theory to estimate the energy shifts to each state in part (a). You may not need to separately compute the shift for every state if a group of states (e.g., a spin triplet) all shift by the same amount, but your answer should clearly indicate what happens to all states.

3) Two identical spin-1/2 fermions are bound by a one-dimensional harmonic oscillator potential, so that:
\[
H_0 = \frac{\vec{p}_1^2}{2m} + \frac{\vec{p}_2^2}{2m} + \frac{m\omega^2}{2} X_1^2 + \frac{m\omega^2}{2} X_2^2.
\]
(a) What are the ground and first excited states and their energies? (b) A perturbation, \( H' = \alpha \vec{S}_1 \cdot \vec{S}_2 \delta(X_1 - X_2) \) is added. Compute the first-order energy shift for the ground and first excited states.

4) Two attractive delta-function potentials in one dimension are separated by a distance \( D \), so that
\[
H = \frac{\vec{p}^2}{2m} - g \delta(x + D/2) - g \delta(x - D/2).
\]
(a) Use the variational wave function
\[
\psi(x) = N \left[ \exp\left(-|x + D/2|/a\right) + \exp\left(-|x - D/2|/a\right) \right]
\]
to put an upper bound on the ground state energy. (b) Choose another trial state, place a second bound and compare to (a).

5) Assume \( H_0 \) is
\[
H_0 = \frac{\vec{p}^2}{2m} - \frac{\alpha}{r^3},
\]
the hydrogen hamiltonian, which has ground state energy \( E_1 \). Add a perturbation \( H' = gz \) and compute the first-order energy shifts to the ground and first excited states, \( n = 1 \) and \( n = 2 \).