PHYSICS 633 Spring 2007 Exam #3 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. This exam covers material from the entire Spring quarter. Some of the examples below have appeared in homeworks, exams and sample exams given earlier this quarter.

1) $H_0 = -\omega_0 S_z$, $H_1 = \omega_1 S_x\, \theta(t)\, e^{-t/\tau}$. If a particle starts in its ground state, what is the probability that its spin will flip in leading order time-dependent perturbation theory as $t \to \infty$?

2) $V(x) = \delta x^4$. Use the trial state $\psi(x) = e^{-ax^2}$ to find an upper bound on the ground state energy. Plot a as a function of δ and discuss why it is reasonable, e.g. why it behaves as it does for very small δ and very large δ.

3) Use the Born approximation to estimate (a) the differential cross section $\frac{d\sigma}{d\Omega}$ as a function of the scattering angle and (b) the total cross section for a Yukawa potential, $V(r) = -V_0\, \frac{e^{-\mu r}}{r}$. Assume the energy is $E = \frac{k^2}{2m}$.

4) Compute the s-wave phase shift, δ_0, for $V(r) = V_0\, \theta(a - r)$. (a) Use this phase shift to approximate the total cross section as the energy goes to zero. (b) Let $E = V_0$ (not necessarily small) and estimate the total cross section as $a \to 0$. Note that zero is not a good approximation for anything that is not absolutely zero. Assume the energy is $E = \frac{k^2}{2m}$.

5) $H_0 = \frac{P^2}{2m} - \frac{\alpha}{r}$, the hamiltonian for hydrogen. At $t = 0$ the proton's charge is turned off and slightly later at $t = \tau$ it is turned back on. Find $H_1(t)$ that will do this and compute the probability that a hydrogen atom starting in its ground state will make a transition to one of the $n = 2$ excited states in leading order time-dependent perturbation theory.

6) Two identical spin-1/2 fermions are confined to a 2-dimensional box, $0 < x < a$ and $0 < y < a$. (a) Compute the energies of the ground state and first excited state and their degeneracies (i.e., how many different states have that energy). (b) A "small" perturbative potential $V(\mathbf{r_1}, \mathbf{r_2}) = \alpha\, \delta^2(\mathbf{r_1} - \mathbf{r_2})$ is added to the potential that confines these fermions to a box. Use first-order perturbation theory to estimate the energy shifts to each state in part (a). You may not need to separately compute the shift for every state if a group of states (e.g., a spin triplet) all shift by the same amount, but your answer should clearly indicate what happens to all states.

7) Use the Born approximation to estimate the total cross-section, σ, for $V(r) = \alpha\ \delta(a - r)$.

8) Two identical spin-1/2 fermions are bound by a one-dimensional harmonic oscillator potential, so that:

$$H_0 = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{m\omega^2}{2} X_1^2 + \frac{m\omega^2}{2} X_2^2.$$

(a) What are the ground and first excited states and their energies? (b) A perturbation, $H' = \alpha \mathbf{S_1} \cdot \mathbf{S_2} \delta(X_1 - X_2)$ is added. Compute the first-order energy shift for the ground and first excited states.

9) Use the $l = 0$ partial wave only, computing the phase shift for a **repulsive** potential $V(r) = V_0 \theta(a - r)$. Assume the energy is less than V_0, so that the potential is classically "forbidden". Given δ_0, what is the cross section for $E = k^2/(2m)$ when $E < V_0$, and what is the limit as $k \to 0$?

10) Use the $l = 0$ partial wave only, computing the $l = 0$ phase shift for a **repulsive** potential $V(r) = V_0 \theta(a - r)$. Assume the energy is **greater** than V_0. After finding δ_0, estimate the cross section for $E = k^2/(2m)$ when $E > V_0$ and find the limit as $E \to V_0$?

11) Use the Born approximation to estimate (a) the differential cross section $\frac{d\sigma}{d\Omega}$ as a function of the scattering angle and (b) the total cross section for a potential, $V(r) = V_0\ \frac{a\ \theta(a-r)}{r}$. Assume the energy is $E = \frac{k^2}{2m}$. (c) Look at the limits $ka << 1$ and $ka >> 1$. If there are simplifications comment and discuss why this behavior might be expected or why it is irrelevant because the Born approximation is simply breaking down.

12) Two attractive delta-function potentials in one dimension are separated by a distance D, so that

$$H = \frac{P^2}{2m} - g\ \delta(x + D/2) - g\ \delta(x - D/2).$$

(a) Use the variational wave function

$$\psi(x) = N \Big[exp[-|x + D/2]/a] + exp[-|x - D/2|/a] \Big]$$

to put an upper bound on the ground state energy. (b) Choose another trial state, place a second bound and compare to (a).

13) Assume H_0 is

$$H_0 = \frac{\mathbf{P^2}}{2m} - \frac{\alpha}{r},$$

the hydrogen hamiltonian, which has ground state energy E_1. Add a perturbation $H' = gz$ and compute the first-order energy shifts to the ground and first excited states, $n = 1$ and $n = 2$.