PHYSICS 633 Spring 2006 Third Midterm

Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Use the Born approximation to estimate (a) the differential cross section $\frac{d\sigma}{d\Omega}$ as a function of the scattering angle and (b) the total cross section for a potential, $V(r) = V_0 \, \frac{a \, \theta(a-r)}{r}$. Assume the energy is $E = \frac{k^2}{2m}$. (c) Look at the limits $ka << 1$ and $ka >> 1$. If there are simplifications comment and discuss why this behavior might be expected or why it is irrelevant because the Born approximation is simply breaking down.

2) Two identical spin-1/2 fermions are confined to a 1-dimensional box. (a) Compute the energies of the ground state and first excited state and their degeneracies (i.e., how many different states have that energy). (b) A "small" perturbative potential $V(x_1, x_2) = \alpha \, \delta^2(x_1 - x_2)$ is added to the potential that confines these fermions to a box. Use first-order perturbation theory to estimate the energy shifts to each state in part (a). You may not need to separately compute the shift for every state if a group of states (e.g., a spin triplet) all shift by the same amount, but your answer should clearly indicate what happens to all states.

3) A relativistic spin-1/2 particle satisfies the Dirac equation. A step function mass is set up so that $M(z) = m + \mu\theta(z)$. Compute the probability that a positive energy spin up fermion with $p_x = p_y = 0$ will be reflected from or will transmit through this step function mass. Fix the energy E and consider both the $E >> m + \mu$ and the $E << m + \mu$ limits.