Dirac Eq.

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \left(c^2 \hat{P}^2 + m^2 c^4\right)^{1/2} |\psi\rangle$$

↳ very asymmetric in space & time

$$E^2 = \vec{p}^{\,2} + m^2$$

$$-\frac{\partial^2}{\partial t^2} |\psi\rangle = \left(\hat{P}^2 + m^2\right) |\psi\rangle$$

$$\rightarrow \left(-\nabla^2 + m^2\right) |\psi\rangle$$

$$\left(\Box + m^2\right) |\psi\rangle = 0$$

or:

$$\sqrt{\hat{P}^2 + m^2} \rightarrow \sqrt{\left(\vec{\alpha} \cdot \vec{P} + \beta m\right)^2}$$

need $\left(\vec{\alpha} \cdot \vec{P} + \beta m\right)^2 = \left(\vec{\alpha} \cdot \vec{P}\right)^2 + \left(\vec{\alpha}\beta + \beta\vec{\alpha}\right) \cdot m\vec{P}$

$$+ \beta^2 m^2$$

$$= \vec{P}^2 + m^2$$

$$\Rightarrow \quad \alpha_i^2 = \beta^2 = 1$$

$$\alpha_i \alpha_j + \alpha_j \alpha_i = 0 \quad \text{if } i \neq j$$

$$\alpha_i \beta + \beta \alpha_i = 0$$

$$ab + ba \equiv \{a, b\} \text{ - anticommutator}$$

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- each element is a 2×2 matrix

$$\Rightarrow \quad i\hbar \frac{\partial}{\partial t} |\psi\rangle = \left(c \vec{\alpha} \cdot \vec{P} + \beta mc^2\right) |\psi\rangle$$

Dirac Equation