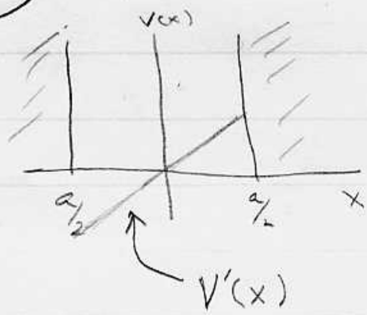
Physics 633 SP05 Sample MT#1 Probs

(1) Particle in a box from $-a/2$ to $a/2$.



$$\phi_n^{(0)} = \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \quad , n \text{ odd}$$

$$= \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad , n \text{ even}$$

$$E_n^{(0)} = n^2 \epsilon \quad , \epsilon = \frac{\hbar^2 \pi^2}{2ma^2}$$

Add the perturbation $V'(x) = \alpha x$.

i) What is $E_n^{(1)}$?

ii) Write $E_n^{(2)}$ as a sum & define the matrix elements that appear as integrals. Evaluate the integral.

(2) Same problem, but now:

$$H' = \alpha x \sin(\omega t)\,\Theta(t)$$

What is the transition probability for gd. state to $\phi_n^{(0)}$, $P_n(t) \simeq |c_n^{(1)}(t)|^2$, in first order time-dependent perturbation theory.

(3) $$H_0 = \frac{P_r^2}{2m} + \frac{L^2}{2mr^2} - \frac{e^2}{r}$$ – Coulomb hamiltonian

$$H' = \alpha^2 \delta^3(\vec{r})\, \vec{S}_{proton} \cdot \vec{S}_{electron}$$

$$E_n^{(0)} = -\frac{1}{n^2}\mathcal{R}, \quad \mathcal{R} = \text{Rydberg} \sim 13.6\,\text{eV}$$

$\alpha = \frac{1}{137}$, so the perturbation is weak

Compute $E_n^{(1)}$ for $n=1$ and $n=2$.

(4) Particle in a sphere, $r<a$.

$$H' = \beta\,\delta^3(\vec{r})\,\exp\left[-\frac{|t|}{\tau}\right].$$

slowly turns on & off

Assume the particle is in the gd. state at $t=-\infty$. What is the probability it will be in state $|n l m\rangle$ at $t\to\infty$?

(5) Harmonic Oscillator $H = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right)$

$$H'(t) = \beta\left(a^\dagger + a\right)\left[2\cos(\omega t)\right]\Theta(t)$$

Start in the gd. state, $|0\rangle$, and compute:

$P_n(t) = |\langle n|\psi(t)\rangle|^2$ in 1st order time-dep. pert. th.