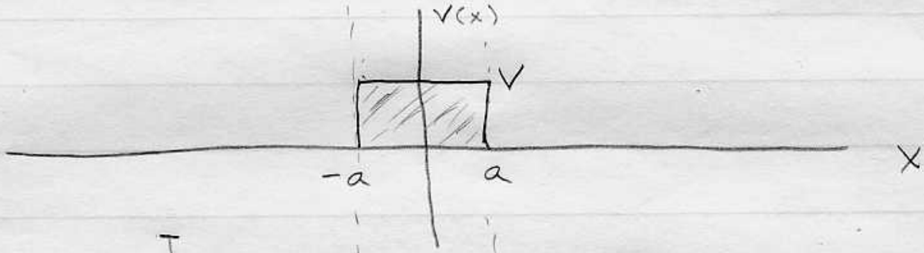
Potential Barrier



I $\qquad$ II $\qquad$ III

$\phi_{I} \qquad \phi_{II} \qquad \phi_{III}$

tunneling: $E<V$

$$E = \frac{k^2}{2m} = -\frac{\kappa^2}{2m} + V \quad \left\{ \begin{array}{c|c|c} \xrightarrow{e^{ikx}} & Ce^{\kappa x} & Fe^{ikx} \\ \xleftarrow{Be^{-ikx}} & De^{-\kappa x} & \longrightarrow \end{array} \right.$$

transmission + reflection, $E > V$

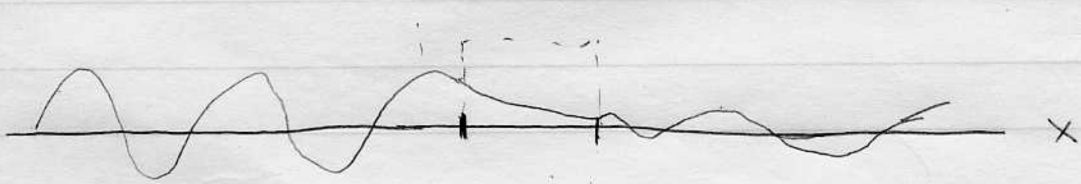
$$E = \frac{k^2}{2m} = \frac{p^2}{2m} + V \quad \left\{ \begin{array}{c|c|c} \xrightarrow{e^{ikx}} & Ce^{ipx} & Fe^{ikx} \\ \xleftarrow{Be^{-ikx}} & De^{-ipx} & \longrightarrow \end{array} \right.$$

$$T = |F|^2$$

$$R = |B|^2$$

Expect:

$E<V$



$E>V$

$E < V$

$$x=-a \begin{cases} e^{-ika} + Be^{ika} = Ce^{-\kappa a} + De^{\kappa a} & (1) \\ ik\left(e^{-ika} - Be^{ika}\right) = \kappa\left(Ce^{-\kappa a} - De^{\kappa a}\right) & (2) \end{cases}$$

$$x=a \begin{cases} Ce^{\kappa a} + De^{-\kappa a} = Fe^{ika} & (3) \\ \kappa\left(Ce^{\kappa a} - De^{-\kappa a}\right) = ikFe^{ika} & (4) \end{cases}$$

details } rewrite: (2') $\frac{ik}{\kappa}\left(e^{-ika} - Be^{ika}\right) = Ce^{-\kappa a} - De^{\kappa a}$ $\boxed{2'}$

add (1) + (2'): $2Ce^{-\kappa a} = \left(1+\frac{ik}{\kappa}\right)e^{-ika} + \left(1-\frac{ik}{\kappa}\right)Be^{ika}$ $\boxed{5}$

subtract (1) + (2'): $2De^{\kappa a} = \left(1-\frac{ik}{\kappa}\right)e^{-ika} + \left(1+\frac{ik}{\kappa}\right)Be^{ika}$ $\boxed{6}$

Next do the same thing with (3) + (4)

$$2Ce^{\kappa a} = \left(1+\frac{ik}{\kappa}\right)Fe^{ika} \qquad \boxed{7}$$

$$2De^{-\kappa a} = \left(1-\frac{ik}{\kappa}\right)Fe^{ika} \qquad \boxed{8}$$

$$\frac{(7)}{(8)} \Rightarrow \frac{C}{D}e^{2\kappa a} = \frac{\kappa+ik}{\kappa-ik}$$

$$D = \left(\frac{\kappa-ik}{\kappa+ik}\right)e^{2\kappa a}\,C \qquad \boxed{8'}$$

Use (5) + (6) with (7) + (8)

$$\boxed{9} \quad \left(1+\frac{ik}{\kappa}\right)e^{-ika} + \left(1-\frac{ik}{\kappa}\right)B e^{ika} = \left(1+\frac{ik}{\kappa}\right)F e^{ika-2\kappa a}$$

$$\boxed{10} \quad \left(1-\frac{ik}{\kappa}\right)e^{-ika} + \left(1+\frac{ik}{\kappa}\right)B e^{ika} = \left(1-\frac{ik}{\kappa}\right)F e^{ika+2\kappa a}$$

$$\boxed{9'} \quad F = e^{-2ika+2\kappa a} + \left(\frac{\kappa-ik}{\kappa+ik}\right)B e^{2\kappa a}$$

$$\boxed{10'} \quad = e^{-2ika-2\kappa a} + \left(\frac{\kappa+ik}{\kappa-ik}\right)B e^{-2\kappa a}$$

subtract $\quad e^{-2ika}\left(e^{2\kappa a} - e^{-2\kappa a}\right)$

$$= -\left[\left(\frac{\kappa-ik}{\kappa+ik}\right)e^{2\kappa a} - \left(\frac{\kappa+ik}{\kappa-ik}\right)e^{-2\kappa a}\right]B$$

$$B = -e^{-2ika}\frac{2\sinh(2\kappa a)}{2\left(\frac{\kappa^2-k^2}{\kappa^2+k^2}\right)\sinh(2\kappa a) - \left(\frac{4i\kappa k}{\kappa^2+k^2}\right)\cosh(2\kappa a)}$$

$$\boxed{B = -e^{-2ika}\left[\left(\frac{\kappa^2-k^2}{\kappa^2+k^2}\right) - 2i\frac{\kappa k}{\kappa^2+k^2}\coth(2\kappa a)\right]^{-1}}$$

$$\mathcal{R} = |B|^2 = \left[\left(\frac{K^2-k^2}{K^2+k^2}\right)^2 + 4\left(\frac{Kk}{K^2+k^2}\right)^2 \coth^2(2Ka)\right]^{-1}$$

$\coth^2 = 1 + \frac{1}{\sinh^2}$

$$\boxed{\mathcal{R} = \left[1 + 4\left(\frac{Kk}{K^2+k^2}\right)^2 \left(\frac{1}{\sinh^2(2Ka)}\right)\right]^{-1}}$$

This is rewritten using : $2mE = k^2$; $2m(E-V) = -K^2$

$$\mathcal{R} = \left\{1 + 4\,\frac{E(V-E)}{V^2}\left(\frac{1}{\sinh^2(2Ka)}\right)\right\}^{-1}$$

$$F = e^{-2ika+2Ka} - \left(\frac{K-ik}{K+ik}\right) e^{-2ika+2Ka}\, \frac{e^{2Ka} - e^{-2Ka}}{\left(\frac{K-ik}{K+ik}\right)e^{2Ka} - \left(\quad\right)e^{-2Ka}}$$

$$= e^{-2ika+2Ka}\left\{1 - \frac{e^{2Ka} - e^{-2Ka}}{e^{2Ka} - \left(\frac{K+ik}{K-ik}\right)^2 e^{-2Ka}}\right\}$$

$$= e^{-2ika+2Ka}\left\{\frac{\cancel{e^{2Ka}} - (\quad)^2 e^{-2Ka} - \cancel{e^{2Ka}} + e^{-2Ka}}{e^{2Ka} - (\quad)^2 e^{-2Ka}}\right\}$$

$$F = e^{-2ika} \frac{1 - \left(\frac{\kappa + ik}{\kappa - ik}\right)^2}{e^{2\kappa a} - \left(\text{''} \right)^2 e^{-2\kappa a}}$$

$$= e^{-2ika} \frac{-4i\kappa k}{(\kappa - ik)^2 e^{2\kappa a} - (\kappa + ik)^2 e^{-2\kappa a}}$$

$$= e^{-2ika} \frac{-4i\kappa k}{2(\kappa^2 - k^2)\sinh(2\kappa a) - 4i\kappa k \cosh(2\kappa a)}$$

$$\boxed{F = \frac{e^{-2ika}}{\cosh(2\kappa a) + i \frac{(\kappa^2 - k^2)}{2\kappa k} \sinh(2\kappa a)}}$$

$$T = |F|^2 = \left\{ \cosh^2(2\kappa a) + \frac{(\kappa^2 - k^2)^2}{4\kappa^2 k^2} \sinh^2(2\kappa a) \right\}^{-1}$$

$\cosh^2 = 1 + \sinh^2$

$$T = \left\{ 1 + \frac{(\kappa^2 + k^2)^2}{4\kappa^2 k^2} \sinh^2(2\kappa a) \right\}^{-1}$$

$$= \left\{ 1 + \frac{V^2}{4E(V-E)} \sinh^2\left(2\sqrt{2m(V-E)}\, a\right) \right\}^{-1}$$