PHYSICS 632 Winter 2007 Exam #1 Sample Problems

Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer.

1) Rigid Rotor in a Magnetic Field. $H = \frac{L^2}{2I} + \mu B L_z$. To simplify the problem, we will adjust B so that $\mu B = 3/I$, which gives us:

$$H = \frac{1}{2I}\Big[L^2 + 6L_z\Big]$$

(a) Find the ground state(s).

(b) $|\psi(t=0)> = \frac{1}{\sqrt{2}}\Big[|1,-1> - |1,1>\Big]$

If L_x is measured at time t_1, what are the possible results and their probabilities?

(c) If the measurement at time t_1 yields zero for L_x, and L_z is measured at time t_2, what are the possible results and their probabilities?

2) Particle Confined to a Sphere of Radius R. At time $t = 0$, $|\psi(\mathbf{r}, t = 0 >= N * (R - r)$ for $r < R$ and zero for $r > R$.

(a) Find N by normalizing $|\psi(t = 0)>$.

(b) What is $\psi(\mathbf{r}, t>$? Write your answer as a sum and compute any integrals that appear.

(c) What is $<r>(t) = \int d^3r |\psi(\mathbf{r}, t)|^2 \; r$?

3) Hydrogen Atom. The hydrogen eigenstates are $<\mathbf{r}|nlm> = \phi_{nlm}(\mathbf{r})$, where the energies are $E_n = -\mathcal{R}/n^2$. Any specific states you need are given in the notes. The initial state is $|\psi(t=0)> = \frac{1}{\sqrt{2}}\Big[|100> + |211>\Big]$.

(a) What are $<x>(t)$ and $<z>(t)$?

(b) As a function of time, what is the probability of finding $z > 0$?

4) A particle in a sphere, $r < a$, is in the state:

$$\psi(\mathbf{r}, t = 0) = \frac{1}{\sqrt{2}}[\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})],$$

where ϕ_n are the energy eigenstates, with energies E_n, ordered from lowest eigenvalue to highest. The first state is the $l = m = 0$ ground state. (a) What is the lowest energy that can be measured? Assume mass M and give the answer in terms of M and a. (b) What is $\psi(x, t)$? (c) What is $<\hat{R}>(t)$, where $<r|\hat{R}|f> = rf(r)$. Write your answer in terms of well-defined radial integrals.

5) $\hat{H} = \frac{\hat{L}^2}{2I} + \omega_0 \hat{L}_z$. What are the eigenstates and eigenvalues of $\hat{H}$? At $t = 0$ the particle is in the lowest energy eigenstate with $l = 1$. What is $< \hat{L}_x > (t)$.

6) (a) Compute $[\hat{L}_i, \hat{R}^2]$. (b) If $\hat{\mathbf{L}} = \hat{\mathbf{L}}_\mathbf{1} + \hat{\mathbf{L}}_\mathbf{2}$, what is $[\hat{L}^2, \hat{\mathbf{L}}_\mathbf{1} \cdot \hat{\mathbf{L}}_\mathbf{2}]$? Show your work.

7) For angular momentum one-half there are two eigenstates, $|+>$ and $|->$, where $J^2|\pm> = 3/4|\pm>$ and $J_z|\pm> = \pm 1/2|\pm>$. $H = \omega_0 J_z$. At $t = 0$, $|\psi(t=0)> = \frac{1}{\sqrt{2}}\big(|+> + |->\big)$. What is $|\psi(t)>$? What are the possible measurements of J_x? What are the probabilities of each as a function of time?

8) For angular momentum one-half there are two eigenstates, $|+>$ and $|->$, where $J^2|\pm> = 3/4|\pm>$ and $J_z|\pm> = \pm 1/2|\pm>$. $H = \frac{J^2}{2I} + \omega_0 J_z$. At $t = 0$, we measure J_y and find $-1/2$. At time t we measure J_x. What are the possible measurements of J_x? What is the probability of each?

9) Two spin-1/2 particles are governed by $H = \frac{J^2}{2I} + \omega_0 J_z$, where $J = J_1 + J_2$. At time $t = 0$ the total angular momentum is measured and found to be 0. (a) At time t, J_{1z} is measured. What are the possible results and what is the probability of each? (b) At time $2t$ the total angular momentum is measured, J^2. What are the possible results (follow all possible branches from the measurement at time t) and what is the probability for each result?

10) A hydrogen atom is prepared at time $t = 0$ so that it is in the state $|\psi(t=0)> = \frac{1}{\sqrt{2}}|211> + \frac{1}{\sqrt{3}}|210> + \frac{1}{\sqrt{6}}|21-1>$, where the kets are eigenstates $|nlm>$. (a) What is $|\psi(t)>$? (b) What is the probability that the electron will be found with $z > 0$ if its position is measured at time t? Complete all integrations. (c) If we measure L_x instead of position at time t, what are the possible results and the probability for each?