

Write your name on the test booklet. Do NOT simply write an answer. Give a calculation and/or reasoning that supports your answer. Do all work and write all answers in the test booklet. Circle or clearly delineate all relevant work so that I do not take points off for errors in your scratch work.

1) Two particles are in a two-dimensional well so that  $0 \leq x \leq a$  and  $0 \leq y \leq a$ . (a) If the particles are identical bosons with no "spin," what are the energies of the ground state and first excited state and what is the degeneracy of each of these states? What is the position representation of the first excited state wave-function? If there is more than one possible first excited state, pick one and tell me which one you are picking. (b) Same problem for two identical fermions with no spin. (c) If the identical fermions have spin  $1/2$ , what is the ground state energy and the ground state wave function? Include both the spatial and spin parts of the wave function.

2) Two non-identical spin- $1/2$  particles have  $H = H_0 + H'$  with  $H_0 = \omega_0 \mathbf{S}_1 \cdot \mathbf{S}_2 + \omega_1 S_z$ , where  $\omega_0 > 0$  and  $\omega_1 > 0$ , and  $H' = \omega_2 S_{1z}$ . (a) What are the eigenstates of  $H_0$  and their energies? (b) Treating  $H'$  as a perturbation compute the first order shifts in the energies of each zeroth-order eigenstate. (c) Compute the second-order shift in the energy of the ground state.