Angular Momentum   Orbital → L , Spin → S

Let J stand for L or S (j for l or s)

$$[J_x, J_y] = i J_z \text{, etc.} \qquad [J^2, J_i] = 0$$

$$J_\pm = J_x \pm i J_y \quad \text{- raises or lowers } m$$

$$J^2 |j, m\rangle = j(j+1) |j, m\rangle \qquad J_z |j,m\rangle = m |j,m\rangle$$

$$J_\pm |j, m\rangle = \sqrt{j(j+1) - m(m\pm 1)}\, |j, m\pm 1\rangle$$

For orbital : $l = 0, 1, 2, \ldots$; $m = -l, -l+1, \ldots, l$

For spin : $s = 0, \frac{1}{2}, 1, \ldots$; $m = -s, -s+1, \ldots, s$

Orbital : $\langle \theta, \phi | l m \rangle = Y_l^m(\theta, \phi)$

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \; ; \; Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}} \cos\theta \; , \; Y_1^{\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\phi}$$

Spin $\frac{1}{2}$: $|\frac{1}{2}, \frac{1}{2}\rangle = |+\rangle$; $|\frac{1}{2}, -\frac{1}{2}\rangle = |-\rangle$

$$|+\rangle \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; |-\rangle \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S^2 \to \frac{3}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \; S_z \to \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \; S_x \to \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hydrogen Atom $\quad H = \frac{P^2}{2m} - \frac{\alpha}{r} = \frac{P_r^2}{2m} + \frac{L^2}{2mr^2} - \frac{\alpha}{r}$

$$H|n\ell m\rangle = \left(-\tfrac{1}{2}\alpha^2 m\right)\frac{1}{n^2}|n\ell m\rangle = -\frac{\mathcal{R}}{n^2}|n\ell m\rangle$$

$\mathcal{R}$ is a Rydberg

For each n, $\ell \geq n+1$

$$\langle\vec{r}|n\ell m\rangle = R_{n\ell}(r)\, Y_\ell^m(\theta,\phi)$$

$$= \psi_{n\ell m}(\vec{r})$$

$$\psi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} = \frac{2}{\sqrt{a^3}} e^{-r/a}\, Y_0^0(\theta,\phi)$$

where $a = \frac{1}{\alpha m}$ - Bohr radius

$$R_{20}(r) = \frac{1}{\sqrt{2a^3}}\left(1 - \frac{1}{2}\frac{r}{a}\right)e^{-r/2a}$$

$$R_{21}(r) = \frac{1}{\sqrt{24a^3}}\frac{r}{a}\, e^{-r/2a}$$